A Thesis on

# Modeling and Simulation of

# Multiple Effect Evaporator System

*submitted by:*


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## CERTIFICATE

This is to certify that the thesis titled "**Modeling and Simulation of Multiple Effect Evaporator System**", submitted to the National Institute of Technology Rourkela by **Ms. G Gautami**, Roll No. **609CH302** for the award of the degree of **Master of Technology (Research)** in Chemical Engineering, is a bona fide record of research work carried out by her under my supervision and guidance. The candidate has fulfilled all the prescribed requirements. The thesis, which is based on candidate's own work, has not been submitted elsewhere for a degree/diploma.

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# ABSTRACT

In the present work a mathematical model based on set of nonlinear equations has been developed for synthesis of multiple effect evaporator (MEE) systems. As evaporator house is one of the most energy intensive units of pulp and paper industries, different configurations are considered in the model to reduce the energy consumption. These are condensate, feed and product flashing, vapor bleeding, steam splitting, etc. Along with these the present model also accounts the complexities of real MEE system such as variable physical properties, boiling point rise. Along with complexities discussed above, the present model also accounts the fouling resistance. For this purpose a linear correlation is developed to predict fouling resistance based on velocity as well as temperature difference. The fouling resistance observed by this correlation is within the limit shown in the literature (Muller-Steinhagen and Branch, 1997). It reduces overall heat transfer coefficient by 11.5% on average.

For the present study two MEE systems of typical Indian pulp and paper industries are considered. First MEE system selected for modeling and simulation is seven effect evaporator system located in north India which is being operated in a nearby Indian Kraft Paper Mill for concentrating weak black liquor using plate falling film evaporators. This system employs steam splitting in first two effects, feed and product flashing along with primary and secondary condensate flashing to generate auxiliary vapor, which are then used in vapor bodies of appropriate effects to improve overall steam economy of the system. The second system used for present study is located in south India. It is ten effect evaporator system used for concentrating

black liquor and being operated in mixed flow sequence. In this system feed and steam splitting as well as vapor bleeding is employed.

For seven effect evaporator system total fourteen models are developed. Initially, a simplest model without any variation is derived based on mass and energy balance. Further, it is improved by incorporating different configurations such as variation in physical properties, BPR, steam splitting, feed, product and condensate flashing and vapor bleeding. These models are developed with and without fouling resistance.

The governing equations of these models are nonlinear in nature. Further, it is observed that for these models the number of equations as well as the number of variables are equal and hence unique solution exist for all cases. The set of nonlinear algebraic equations are solved using software called ‘system of non linear equations’. However, in the present work to incorporate the complex interactions of variables during solution of model an iterative procedure is used.

For seven effect evaporator system total 14 models are proposed to visualize that how individual configuration is affecting the steam economy of the MEE system. The comparison shows that maximum steam economy is observed for the model where flashing as well as vapor bleeding are used. In comparison to the simplest system the improvement in steam economy through best model is found as 27.3%. The modified seven effect evaporator system, obtained using best model, requires four shell and tube heat exchangers and five pumps. This modification has total capital investment as Rs 29.3 lakh. However, saving in steam consumption is found as Rs 21.8 lakh/year thus, total payback period for the modified seven effect evaporator system is 1.3 years. For ten effect evaporator system improvement in steam economy is observed by 12.8% in comparison to existing system. It incorporates three preheaters which use bled vapor from the system. Based on the comparison with published model as well as industrial data it is found that the present model can be effectively applied to simulate the real MEE system and improve the steam economy of MEE system by 15%.

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G.GAUTAMI

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## NOMENCLATURE

| | |
|---|---|
| A | Area of effect, $m^2$ |
| $A_1$, $A_2$ | Solids dependent constants for Eq. 2.21 |
| $A_1,a_2,a_3,a_4$ | Coefficients of cubic polynomial function in Eq. 4.56 |
| A, b, c, d | Coefficients for the Eq. 4.7 |
| $B_1$, $B_2$ | Solids and composition dependent constants for Eq. 2.21 |
| $C_1$ to $C_5$ | Constants for Eqs. 4.4 4.5, 4.57, 4.58, 4.59 and 4.60. |
| C, d | Concentration dependent constants in Equ. 2.5 |
| $C_P$ | Specific heat capacity, kJ/kg/°C |
| F, L | Liquor flow rate, kg/s |
| h | Enthalpy, kJ/kg |
| H | Enthalpy of vapor, kJ/kg |
| P | Product flow rate, kg/s |
| Q | Rate of heat transfer, kW |
| $\lambda$ | Heat of vaporization, kJ/kg |
| R | Fouling resistance, $m^2$°C/kW |
| S | Total solid content in liquor, % |
| T | Vapor body temperature, °C |
| $\tau$ | Boiling point rise, °C |
| U | Overall heat transfer coefficients, kW/$m^2$°C |
| v | Flow rate of bled vapor, kg/s |
| V | Velocity in Eq. 1, elsewhere vapor flow rate, kg/s |
| x | Mass fraction |

*Subscripts*

| | |
|---|---|
| avg | Average |
| 0 | Live steam |
| c | Clean condition |
| d | Fouled condition |
| 1 to 7 k | Effect number |

*Greek letters*

| | |
|---|---|
| $\rho$ | Density, kg/m$^3$ |
| $\mu$ | Viscosity, cP |

*Abbreviations*

| | |
|---|---|
| MEE | Multiple effect evaporator |
| LTV | Long tube vertical |

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**CHAPTER 1**

# INTRODUCTION

Evaporators are integral part of a number of process industries namely Pulp & Paper, Chlor-alkali, Sugar, Pharmaceuticals, Desalination, Dairy and Food processing, etc. The Pulp and Paper industry, which is the focus of the present investigation, predominantly uses the Kraft Process. In this process black liquor is generated as spent liquor. The recovery of valuable chemicals like Sodium Sulfide, Sodium Hydroxide and Sodium Carbonate from it is an integral part of the Kraft Process. It consists of multiple effect evaporator (MEE) system as one of the major section of the recovery process. An energy audit shows that Evaporator House of a Pulp and Paper industry consumes about 24-30% of its total energy designating it as an energy intensive section (Rao and Kumar, 1985).

With the development of falling film evaporator which works under low temperature difference and provides a scope to accommodate more number of evaporators within the maximum value of available $\Delta T$, more and more Indian Pulp and Paper industry have started inducting these evaporators into their MEE system. As falling film evaporators are the latest addition to Indian pulp and paper industry due to its energy efficiency it is an obvious selection for the present study. The energy efficiency of MEE system can be enhanced by inducting feed, product and condensate flashing, feed and steam splitting and vapor bleeding. In the present work two MEE systems of typical pulp and paper industries are considered for analysis based on above configurations. These are seven effect and ten effect evaporator systems.

Over last seven decades mathematical models of MEE systems have been used to analyze these complex systems. Some of these have been developed by Holland (1975), Lambert et al. (1987), El-Dessouky et al. (2000) and Bhagrava et al. (2008). These models are based on set of linear and non-linear equations. Amongst these models Bhargava et al. (2008) proposed a model using generalized cascade algorithm in which model of an evaporator body is solved repeatedly to address different operating configurations of MEE system. However, other investigators proposed equation based models where the whole set of governing equations of the model needs to be changed to address the new operating configuration. These two strategies of modeling are successfully applied to simulate a number of MEE systems.

Mathematical Models which describe the complete process are complex in nature. These models also use complex transport phenomena based mathematical models or empirical models for the prediction of overall heat transfer coefficients (U) of evaporators as a function of liquor flow rate, liquor concentration, physico-thermal properties of liquor and type of evaporator employed. In contrast to these, Khanam and Mohanty (2011) proposed linear model based on principles of process integration. This model worked on the assumption of equal $\Delta T$ in each effect and thus, eliminated the requirement of U in the model.

Though all these models account complexities of real MEE system such as variation in physical properties, feed sequencing, flashing, splitting and bleeding these do not consider the problem associated with fouling condition which is the major problem of MEE system. The efficiency of MEE systems is also affected by fouling and Pulp and Paper industry is one of the worst sufferers of fouling caused by deposition of organic and inorganic materials such as fibers, salts, lignin, flakes etc. on the evaporator surface.

Thus, under the above backdrop it appears that there is a scope for development of mathematical model which can accommodate different operating configurations along with fouling conditions such as variation in enthalpy and latent heat of vaporization, boiling point rise, feed and steam splitting, provision for condensate, feed and product flashing, vapor bleeding. Based on the above discussions the present work has been planned with following objectives:

1. To develop empirical model for computing fouling resistance based on primary parameters such as surface temperature and velocity of liquor.
2. To develop model for different operating configurations such as variation in physical properties of liquor, condensate and vapor, boiling point rise, feed and steam splitting, condensate, feed and product flashing and vapor bleeding with or without fouling resistance.
3. To compare the steam economy predicted by models with or without fouling conditions. Further, to compare results of the present model with that of published models developed for different MEE systems to show the effectiveness and reliability of this model.

# CHAPTER 2

## LITERATURE REVIEW

A thorough literature review on different aspects of multiple effect evaporator (MEE) system has been reported in this Chapter. Though research on evaporator has started in the year 1845, it appears that the first papers related to mathematical modeling of MEE system appeared only in the year 1928 (Badger, 1928). Since then, many investigators have published on different aspects of it such as mathematical modeling, design, operation, cleaning, optimal number of effects, optimal feed flow sequence, etc. As the present work is related to the development of model for the synthesis of MEE system and requires comparison of its results with that obtained from rigorous mathematical simulation, a literature review covering all aspects of the present work is presented in this Chapter. As all simulation and design work utilizes the physical properties of fluids, which is black liquor in this case, literature related to estimation of physico-thermal properties of black liquor has also been incorporated. Finally, literature review for solution techniques to be used for solving a set of simultaneous algebraic equations developed during the modeling is also integrated.

### 2.1 EVAPORATOR AND ITS TYPES

In simple terms evaporation is the process of concentrating a solution to remove the excess of solvent (water in most cases) so as to obtain a final product rich in solute concentration. And an evaporator is used to carry out this process by using steam as a heating medium in most of the cases.

The concept of evaporator body was first introduced by an African-American engineer Norbert Rillieux in 1845. However, the mathematical modeling for its design started in 1928 with the work of Badger. Since then, many investigators have proposed mathematical models for evaporators.

In practice different types of evaporators are used which are often classified as:

- heating medium separated from evaporating liquid by tubular heating surfaces,
- heating medium confined by coils, jackets, double walls, flat plates, etc.,
- heating medium brought into direct contact with evaporating liquid, and
- heating with solar radiation

Based on these classifications following types of evaporators are developed: horizontal tube evaporator, short tube vertical evaporator, long tube vertical evaporator, falling film evaporator, rising film evaporator, forced circulation evaporator, plate evaporator, mechanical aided evaporator, etc. Some of these are discussed below in detail.

In the horizontal-tube evaporator heating tubes are arranged in a horizontal bundle immersed in the liquid at bottom of the cylinder. Liquid circulation is poor in this type of evaporator. By using vertical tubes, rather than horizontal, the natural circulation of the heated liquid is used to provide better heat transfer. In vertical evaporators recirculation of the liquid occurs through a large "downcomer" so that the liquors rise through the vertical tubes about 5-8 cm in diameter. Liquor boils in space just above the upper tube plate and recirculates through the downcomers. The hydrostatic head reduces boiling on the lower tubes, which are covered by the circulating liquid.

The liquid may either pass down through the tubes, called a falling- film evaporator, or be carried up by the evaporating liquor in which case it is called a climbing-film evaporator. Evaporation occurs on the walls of the tubes. Because circulation rates are high and the surface films are thin, good conditions are obtained for the concentration of heat sensitive liquids due to high heat transfer rates and short heating times.

Generally, if the liquid is not recirculated and sufficient evaporation does not occur in one pass, the liquid is fed to another pass. In the climbing-film evaporator as the liquid boils on the inside of the tube slugs of vapor form and this vapor carries up the remaining liquid, which continues to boil. Tube diameters are of the order of 2.5 to 5 cm and contact times may be as low as 5-10 seconds.

The plate heat exchanger can be adapted for use as an evaporator. Spacing between the plates can be increased so that much larger volume of the vapors, when compared with the liquid, can be accommodated. Plate evaporators can provide good heat transfer and also ease of cleaning.

With the advancement of technology several authors reported a shift from climbing/rising film evaporator to falling film evaporator. Fosberg and Claussen (1982) showed that use of vertical tube falling film evaporator provided stable operations at low heat transfer values with high heat transfer rates and permitted more energy efficient evaporation as compared to other type of evaporators. Logsdon (1983) discussed the trend in evaporator applications for the pulp and paper industry which indicated increasing use of plate type free falling film evaporator system. Shalansky et al. (1992) discussed the design features and operating performance of Rosenblad plate type falling film evaporator system. The operational and mechanical problems encountered during initial operation and their solutions were also discussed. Reddy et al. (1992) and Dangwal et al. (1998) discussed their experiences in switching over from long tube vertical evaporators to falling film evaporators in Indian Pulp and Paper industries. It was indicated that falling film evaporators consumed less power and segregated condensate so that 96% of condensate is odor free. A higher steam economy and lesser chemical loss during boiling were indicated. In initial stages modeling of falling film evaporation in vertical tube was studied. Barba and Felice (1984)

Bertuzzi et al. (1985) and Shmerler and Mudawwar (1988) presented various aspects of evaporation of falling liquid films in vertical tubes. Burris and Howe (1987) described the operational experiences of first use of eight-effect tubular falling film evaporator train.

With the invention of flat (plate type) falling film evaporators attention shifted to the study of heat and mass transfer on falling liquid film over a vertical plate. Chuan-bao et al. (1998) in their paper presented a study of falling film on vertical plate having sinusoidal surface and dimple shaped surface for the concentration of black liquor from wheat straw pulp. Meng and Hsieh (1995) carried out pilot plant and scale studies on falling film black liquor evaporator using dimpled plate as heating element. A preliminary experimental study of falling film heat transfer on a vertical double fluted plate is presented by Jin et al. (2002) for the desalination plant. Stefanov and Hoo (2003) proposed a distributed parameter model of black liquor falling film evaporator for a single plate.

## 2.2 MATHEMATICAL MODELING OF A MULTIPLE EFFECT EVAPORATOR SYSTEM

The literature shows that for simulation of a MEE system generally two approaches are employed. First is related to the formulation of set of equations for a MEE system based on the provisions used for that MEE system whereas, the second approach uses a cascade simulation algorithm in which a single effect model is solved many a times for different input sets depending upon the flow sequence and other provisions of the MEE system. Both approaches have their relative merits and demerits. Total number of equations in a model of MEE system depends on the number of equations involved in the user model of an evaporator body. It is interesting to note that number of equations per evaporator body may vary between 4 to 1 depending upon the approach employed for modeling.

### 2.2.1 Model for simple MEE system

Badger and McCabe (1936), Kern (1950), Holland (1975), Nishitani and Kunugita (1979), Zain and Kumar (1996), Agarwal (1992) and Agarwal et al. (2004) proposed models for the simulation of a triple effect evaporator (TEE) system whereas, Leibovic (1958) developed a model for double effect evaporator system. Kern (1950) and Leibovic (1958) used forward feed flow sequence for concentration of chemical solution without BPR and solved a set of equations simultaneously to get the output parameters. Badger and McCabe (1936) studied the above system and solved it iteratively. Holland (1975), Zain and Kumar (1996), Agarwal (1992) and Agarwal et al. (2004) investigated the evaporation of water from caustic soda solution using forward and mixed feed flow sequence. It was interesting to note that Holland (1975), Agarwal (1992) and Agarwal et al. (2004) used a set of twelve equations in their models whereas; Zain and Kumar (1996) proposed only five equations for the simulation of a TEE system. They solved their model equations which were of nonlinear nature using Newton's method. Agarwal (1992) and Agarwal et al. (2004) extended their work to four and five effect evaporator systems for the concentration of sugar and black liquor solutions respectively.

Further, Nishitani and Kunugita (1979) studied a TEE system, with forward, backward and mixed sequences for concentrating milk. The model was formed using a set of twelve equations and was solved iteratively.

Gupta (1986), Tyagi (1987) and Lambert et al. (1987) developed three different mathematical models for three different five effect evaporator systems used for concentrating black liquor using forward sequence, black liquor using mixed sequence and aqueous sodium hydroxide solutions using backward sequence. The model proposed by Gupta (1986) and Tyagi (1987) were solved using dynamic programming and iterative method, respectively, whereas, the model developed by Lambert et al. (1987) contained a set of twenty equations for the above system that was solved employing Gaussian elimination method.

Mathur (1992) and Mariam (1998) studied a sextuple effect evaporator system for the concentration of black liquor in a pulp and paper mill. Mathur (1992) used backward and mixed sequence and incorporated feed and steam splitting into his model whereas; Mariam (1998) included backward sequence.

It has also been observed that model equations developed for a given operating configuration need to be changed completely when the configuration changes. This further adds difficulty in handling all the operating configurations through a single model without changing the set of its governing equations.

To eliminate the difficulty of equation based models Stewart and Beveridge (1977) proposed the concept of cascade simulation for a MEE system. This is basically a solution technique that can incorporate any user defined model developed for an evaporator body. In fact, it solves the model of an evaporator body 'n' times per iteration in a predetermined sequence decided by the feed flow sequence and operating configuration of a MEE system. Thus, it reduces the number of equations which should be solved simultaneously to arrive at a solution. Moreover, with change in operating configuration the sequence of solution of model equation of effect changes. The solution strategy automatically selects above sequence based on the input data file where designer describes the operating configuration.

Stewart and Beveridge (1977) solved a TEE system with backward sequence for the concentration of caustic soda using cascade algorithm. Ayangbile et al. (1984) extended the work of Stewart and Beveridge (1977) and proposed a generalized cascade algorithm to solve MEE system. Their algorithm could easily incorporate feed splitting and different feed flow sequences.

### 2.2.2 Model for MEE system with flashing and vapor bleeding

Itahara and Stiel (1966, 1968) employed forward and backward sequences for concentrating saline water in three and eight effect evaporator systems, respectively. Further, they incorporated re-heaters to preheat the liquor using bled vapor. They solved these models using dynamic programming and obtained optimum number of effects for above two cases.

Radovic et al. (1979) developed mathematical models for five effect evaporator system used for concentrating sugar solution using forward sequence. They included condensate flashing and vapor bleeding in their model. The bled vapor was used to preheat the sugar solution. Their model contained twenty equations and was solved using Newton's method.

Ray et al. (1992, 2000 and 2004) and Singh et al. (2001) studied a sextuple effect evaporator system for the concentration of black liquor in a pulp and paper mill. Ray et. al. (1992, 2000 and 2004) employed product and condensate flashing in their model and considered mixed sequence whereas, Singh et al. (2001) used backward sequence with feed splitting. They solved their models using Newton's method. Goel (1995) considered backward sequence along with condensate flashing for the investigation of above system and solved it using Broyden's method.

Bremford and Muller-Steinhagen (1994, 1996) investigated two evaporator systems: one having six effects and the other with seven effects used for the concentration of black liquor using backward sequence. They also incorporated feed-, product- and condensate- flashing, feed and steam splitting and re-heaters in their model and solved it iteratively.

El-Dessouky et al. (2000) developed the model for three different MEE systems having four effects, six effects and twelve effects for desalination process. The authors included condensate flashing in the mathematical model and used forward sequence for all the three cases and solved the developed sets of equations employing Newton's method.

Bhargava (2004) and Bhargava et al. (2008) proposed model for the simulation of septuple effect flat falling film evaporator system for the concentration of black liquor using backward, mixed and Scandinavian feed sequence. For their work, they used model of an evaporator body, which contained only one equation, and used the generalized cascade algorithm for solving this model for a variety of configurations. This included different feed flow sequences, feed- and steam-splitting, feed-, product- and condensate- flashing, vapor bleeding for re-heaters, etc. The authors also developed many empirical correlations such as correlations for BPR, overall heat transfer coefficient of flat falling film evaporator, physical property of black liquor and heat losses from different effects to be incorporated in their model to make it more reliable and close to reality.

In contrast to all these models, Khanam and Mohanty (2011) proposed linear model for septuple effect evaporator system based on principles of process integration. They incorporated many complexities of MEE system such as different feed flow sequences, steam splitting, feed, product and condensate flashing, vapor bleeding, etc. This model worked on the assumption of equal $\Delta T$ in each effect and thus, eliminated the requirement of U in the model.

## 2.3 MODELS WITH CONSIDERATIONS OF FOULING CONDITIONS

It appears that the research on fouling activity started from 1910. Since then it is being studied extensively. One of the commonly practiced methods was to allow extra surface area to compensate for heat loss caused due to fouling, but then this led to the problem involving large area heat exchangers and difficulty in maintaining operability conditions. A few studies on fouling of MEE system are discussed below:

Muller-Steinhagen (1998) dealt with the problem of fouling in industries of Kraft pulping process and Bayer bauxite refining process. It was found that in order to reduce fouling the operation should be below a threshold surface temperature and/or liquor concentration. The use of plate heat exchangers or of PTFE (polytetrafluoroethylene) coated surfaces was also recommended. Muller-Steinhagen and Branch (1997) studied the effect of velocity as well as bulk temperature on fouling rate when evaporation of black liquor was carried out.

Schmidl and Frederick (1999) conducted a survey on evaporator fouling where a collection of samples of black liquor and scales from about 40 Kraft pulping industries and their detailed analyses were performed. The key results of the survey were that the average product solids content from evaporator trains was increased from 49% to 58% due to utilization of falling film technology with liquor recirculation. The average overall heat flux, U and temperature difference per effect were reduced by 10%, 8%, and 3% respectively, compared to the values used two decades ago. Chen and Gao (2004) suggested that lower surface-to-bulk temperature difference and lower surface temperature were reasonable ways to improve both concentration and control of soluble scaling for high-solids-concentration black liquor.

Eneberg et al. (2000) presented a method where calcium carbonate scaling in multi stage evaporation of a Kraft black liquor was reduced by heat treating the liquor for 1-20min at a temperature of 110- 145˚C to reduce the amount of calcium in the liquor.

Euhus et al. (2003) and Frederick et al. (2003) worked on fouling in falling film evaporators during evaporation of black liquor and suggested measures to control soluble scale formation and to eliminate sodium salt fouling.

## 2.4 ENERGY REDUCTION SCHEMES FOR MEE SYSTEM

In literature many investigators used number of energy reduction schemes in their models such as different flow sequences, flashing, bleeding, etc. Khanam and Mohanty (2010) proposed a new energy reduction scheme where condensate of an evaporator was used to preheat the liquor using a counter current heat exchanger. It helped in reducing the steam consumption for a multiple effect evaporator (MEE) system. Khanam and Mohanty (2010) carried out a comparative study between different ERSs such as condensate-, feed- and product- flashing, vapor bleeding and new scheme. These schemes were employed by changing the design of the existing MEE system using units such as flash tanks, heat exchangers, etc. They showed that energy reduction schemes saved steam up to 24.6%. The best scheme was selected based on steam consumption as well as total annual cost involved in the scheme. Further, they extended the work by developing a simplified technique called modified temperature path to reduce steam consumption without modifying the MEE system. This technique was used to select the optimal flow sequence amongst the feasible sequences based on shortest temperature path and U-turns. This approach was easy and needed comparatively less computation in comparison to other techniques based on complex simulation, which were generally used for screening of optimum sequences.

Darwish and El-Dessouky (1996), Minnich et al. (1995) and El-Dessouky et al (1985) developed analytical models where they studied and compared the effects of thermal and mechanical vapor compressions on the MEE system.

## 2.5 PHYSICO – THERMAL PROPERTIES OF BLACK LIQUOR

For the simulation of MEE system used for concentrating black liquor, it is necessary that physico-thermal properties of black liquor such as density and specific gravity, specific heat capacity, viscosity and BPR should be known a priory. These properties of black liquor generally depend on the presence of different organic and inorganic constituents and its concentration in the liquor and temperature of it. The organic compounds include alkali lignin, thiolignin, iso-saccharinic acid, polysaccharides, resin and fatty acids whereas; inorganic compounds contain sodium hydroxide, sodium sulphide, sodium carbonate, sodium sulphate, sodium thiosulphate, sodium polysulphides, elemental sulphur and sodium sulphite. Thus, a brief review of literature on above properties is presented below:

### 2.5.1 Specific Heat Capacity

Regested (1951) proposed a correlation for specific heat capacity of black liquor as a function of total solid content (%S) of the liquid. He considered the dependence of specific heat on the temperature to be negligible.

$$C_{pL} = 4187\ [1\text{-}0.0054\ [\%S]] \tag{2.1}$$

Veeramani (1978, 1982) extended the correlation, proposed by Regested (1951), and developed a correlation for specific heat capacity of bamboo and pine black liquors as:

$$C_{pL} = [1.8\text{x}10^{-3}T_L - 0.54]\ \text{x}\ 10^{-2}\ [\%S] + 1 \tag{2.2}$$

Hultin (1968) proposed the following relationship for specific heat of black liquor:

$$C_{pL} = 0.96 - 0.45\ \text{x}\ 10^{-2}\ (\%S) \tag{2.3}$$

Grace and Malcolm (1989) recommended following expression;

$$C_{pL} = 1\text{-}[1\text{-}C_{ps}]\ \text{x}\ 10^{-2}\ \text{x}\ [\%S] \tag{2.4}$$

Further, Zaman and Fricke (1996) determined the correlation of specific heat of slash pine Kraft black liquor as

$$C_{pL} = 3.98 + 6.19\ \text{x}\ 10^{-4}(T) + (c + d\ T)\ \text{x} \tag{2.5}$$

Where, c and d are concentration-dependent constants that have been correlated to the pulping conditions for the liquors.

### 2.5.2 Boiling Point Rise

The boiling temperature of black liquor is a strong function of solid concentration and a weak function of pressure. The BPR increases more rapidly at higher solid concentration. Hultin (1968) proposed the following expression for BPR as a function of solid concentration:

$$BPR = \frac{K\ (\%S)}{(100 - (\%S))} \qquad (2.6)$$

where, K is a constant and just equivalent to the BPR at 50% solid concentration.

The TAPPI correlation (Ray et al., 1992), employed for computing BPR, is:

$$BPR = 23\ [0.1 + ((\%S)/100)]^2 \qquad (2.7)$$

Zaman and Fricke (1998) studied the BPR of slash pine Kraft black liquors for a wide range of solid concentrations (up to 85%) and proposed the following correlation:

$$BPR=(a_1+b_1Pr)\ [x/(1-x)] \qquad \text{for } x < 0.65 \qquad (2.8)$$

$$BPR=[(a_2+b_2Pr)+(a_3+b_3Pr)]\ [x/(1-x)] \qquad \text{for } x \geq 0.65 \qquad (2.9)$$

Where, $a_1$, $b_1$, $a_2$, $b_2$, $a_3$ and $b_3$ are experimentally determined constants.

### 2.5.3 Density and Specific Heat

Regested (1951) proposed following correlation for the computation of density of black liquor, as a function of solids concentration ($T_s$) and temperature of liquor.

$$\rho = 1007 + 6\ T_s - 0.495\ T_L \qquad (2.10)$$

Hultin (1968) plotted density vs solids concentration of black liquor at a temperature of 90 °C and developed following two linear relationships for two cases:

Case 1: For solids concentration between 10 to 25%:

$$T_s = 177\ (\rho - 963) \qquad (2.11)$$

Case 2: For solids concentration between 50 and 65%

$T_s = 146\ (\rho - 920)$ (2.12)

Further, Koorse et al. (1974, 1975) determined experimentally the specific gravity of black liquor for different concentrations at 70 °C for raw materials such as bamboo, bagasse, eucalyptus, etc. They presented the results in the form of graphs and concluded that the black liquors from eucalyptus, bamboo and bagasse exhibited increasing specific gravity in that order.

### 2.5.4 Viscosity

A group of investigators such as Kobe and McCormack (1949), Passinen (1968), Kim et al. (1981), Venkatesh and Nguyen (1985) and Grace and Malcolm (1989) presented viscosity data of different black liquors as a function of solids concentration and temperature. Koorse et al. (1974, 1975) presented the plots of viscosity of black liquor at 70 °C with the variation of solid concentration for different black liquors produced from raw materials like bamboo, bagasse, eucalyptus and pine. Further, they observed that bagasse black liquor had highest viscosity whereas, pine black liquor exhibited lowest viscosity among the different black liquors investigated. At 45 % solid concentration and 70 °C, viscosity of bagasse black liquor is 10 times that of the value for bamboo black liquor and 100 times the value of viscosity for pine black liquor. These ratios are smaller at lower concentration. Hultin (1968) presented data of kinematic viscosity for some black liquors as functions of temperature and solid concentrations.

The correlations of viscosity of black liquor recommended in TAPPI monograph and cited by Ray et. al. (1985, 1989 and 1992), for various solids content are reproduced below:

For $T_s < 40\%$

$$\mu = \exp [-8.3\text{x}10^{-3} - 6.55 \text{ x } 10^{-3} (T_s/100)^2 + 5.62 \text{ x } 10^{-2} (T_s/100)^2 (T_F - 660)$$
$$+5.7 (T_s/100) - 1.307] \qquad (2.13)$$

and for $T_s > 40\%$

$$\mu = 0.062 \exp [-3.26\text{x}10^{-3} (T_F - 460) + 0.102(T_s) + 37.3 - 6 (T_F-460)^2$$

$$+1.8\text{x}10^{-3} (T_s)^2 + 4.95\text{x}10^{-4} (T_F - 460) (T_s)] \tag{2.14}$$

Based on the experimental data proposed by Kobe and McCormack (1949) and Davis (1955), Gudmundson (1971, 1972) developed the following correlation for the computation of viscosity:

$$\mu = \exp [A + B(T_s) + C (T_s)^2 + D (T_s)^3] \tag{2.15}$$

Where,

$$A = 0.4717 - 0.02472(T) + 0.7059\text{x}10^{-5} (T)^2 \tag{2.16}$$

$$B = 0.06973 - 0.5452 \text{ x } 10^{-3}(T) + 0.1656 \text{ x } 10^{-5} (T)^2 \tag{2.17}$$

$$C = 2.046 \text{ x } 10^{-3} + 3.183 \text{ x } 10^{-5}(T) - 9.761 \text{ x } 10^{-8} (T)^2 \tag{2.18}$$

and, $$D = 5.793 \text{ x } 10\text{-}^{5} - 6.129 \text{ x } 10^{-7}(T) + 1.837 \text{ x } 10^{-8} (T)^2 \tag{2.19}$$

Further, Zaman and Fricke (1994, 1995a, 1995b, 1995c and 1996) studied the effect of pulping conditions and black liquor composition on the viscosity and proposed the following correlations:

$$\mu = A_1 T^{0.5} \exp\left[\frac{B_1}{T}\right] \tag{2.20}$$

and $$\mu = A_2 T^{0.5} \exp\left[\frac{B_2 T_o}{T - T_o}\right] \tag{2.21}$$

where,

$A_1$ and $A_2$ are solids dependent constants.

$T_o$ is reference temperature, where free volume becomes zero in absolute scale T is in absolute temperature

$B_1$ is solids and composition dependent constant related to activation energy for flow, and

$B_2$ is solids and composition dependent constant related to free volume.

## 2.6 METHODS FOR SOLVING A SET OF SIMULTANEOUS ALGEBRAIC EQUATIONS

The solution of the simultaneous algebraic equations depends on the type of equations. A brief review on the models for MEE system, discussed in Section 2.2, shows that generally, two types of models for MEE systems are available in the literature. One that consists of set of simultaneous nonlinear algebraic equations whereas, other includes set of simultaneous linear algebraic equations.

For the solution of simultaneous linear and nonlinear equations developed for the simulation of MEE systems, many investigators such as Hassett (1957), Freund (1963), Militzer (1965) and Wiklund (1968) proposed analytical techniques. On the other hand, many authors used available numerical methods for solving these sets of equations. These numerical methods generally depend on initial guess and it is seen that in MEE systems initial values can be guessed with fair accuracy because of the known boundary of input parameters.

For solving simultaneous nonlinear equations, Paloschi (1988) used Quasi-Newton method whereas, many investigators such as Holland (1975), Radovic et al. (1979), Mathur (1992), Agarwal (1992), Mariam (1998), Ray et al. (1992, 2000 and 2004), El-Dessouky et al. (2000) and Agarwal et al. (2004) employed Newton's method which was quite comprehensive for solving these types of equations. In Newton's method an iterative procedure was used to solve the nonlinear equations. Jacobian matrix of first order derivatives was used to get improvements in values of unknown variables. The resulting system was a set of linear algebraic equations. These authors used Gauss elimination method supplemented with LU decomposition to solve set of linear equations. Lambert et al. (1987) suggested to linearize the set of nonlinear equations to get simultaneous linear equations and subsequently solved these using Gauss elimination method. They claimed that the linear method was faster, much more stable and had more desirable convergence characteristics than a widely used nonlinear method.

**CHAPTER 3**

# PROBLEM STATEMENT

The present investigation deals with the modeling and simulation of MEE system. In this Chapter the MEE systems used for concentrating black liquor and their typical operating parameters are discussed.

## 3.1 THE MEE SYSTEM

A literature review on MEE systems used for concentrating weak black liquor in India, U.S.A. and Scandinavian countries by Britt (1964), Arhippainen (1968) and Ray et al. (1985) suggests that number of effects and liquor flow sequences vary considerably. In U.S.A., the pulp and paper mills generally employ sextuple effect evaporator systems, whereas, quintuple effect evaporator systems are used in Scandinavian. Indian paper mills generally adopt sextuple effect LTV type evaporators. However, the recent trend in India is to use Septuple or more number of effects MEE systems.

For the present study two MEE systems of typical Indian pulp and paper industries are considered. Both systems are located in two opposite end of India. These are used to concentrate black liquor and employed plate falling film evaporators. Two MEE systems are described below:

### 3.1.1 Seven effect evaporator system

The MEE system selected for modeling and simulation is the seven effect evaporator system located in north India which is being operated in a nearby Indian Kraft Paper Mill for concentrating weak black liquor. This system is taken from open literature (Bhargava et al., 2008). The schematic diagram of this system with backward feed sequence is shown in Fig. 3.1. The first two effects of it are considered as finishing effects, which require live steam and the seventh effect is attached to a vacuum unit. This system employs feed and steam splitting, feed and product flashing along with primary and secondary condensate flashing to generate auxiliary vapor, which are then used in vapor bodies of appropriate effects to improve overall steam economy of the system. The base case operating and geometrical parameters for this system are given in Table 3.1 which shows that steam going into first effect is 7°C colder than that into second effect. This is an actual scenario and thus it has been taken as it is during simulation. The plausible explanation is unequal distribution of steam from the header to these effects leading to two different pressures in the steam side of these effects.

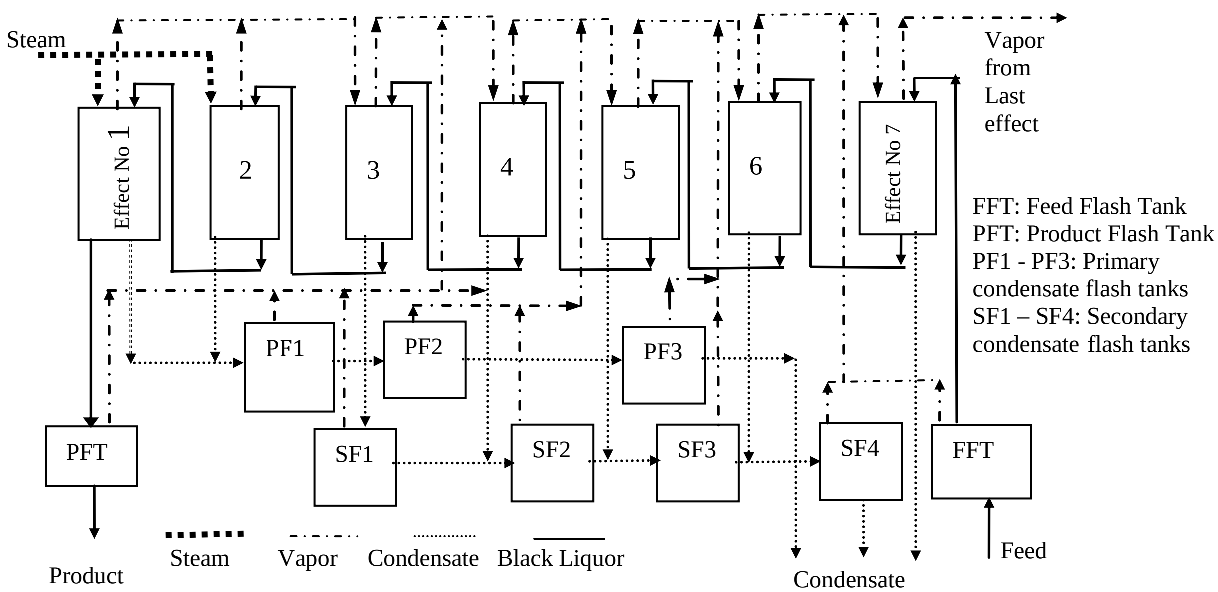



Fig. 3.1 Schematic diagram of seven effect evaporator system

Table 3.1 Typical operating parameters of seven effect evaporator system

| S. No | Parameter(s) | | Value(s) |
|---|---|---|---|
| 1 | Total number of effects | | 7 |
| 2 | Number of effects being supplied live steam | | 2 |
| 3 | Live steam temperature | Effect 1 | 140 °C |
| | | Effect 2 | 147 °C |
| 4 | Black liquor inlet concentration | | 0.118 |
| 6 | Liquor inlet temperature | | 64.7$^{o}$C |
| 7 | Black liquor feed flow rate | | 56200 kg/h |
| 8 | Last effect vapor temperature | | 52 °C |
| 9 | Feed flow sequence | | Backward |
| 10 | Heat Transfer Area | Effect 1 and 2 | 540 m$^2$ each |
| | | Effect 3 to 6 | 660 m$^2$ each |
| | | Effect 7 | 690 m$^2$ |

### 3.1.2 Ten effects evaporator system

The second system used for present study is located in south India. It is ten effect evaporator system used for concentrating black liquor. The schematic diagram of the system with mixed feed flow sequence is shown in Fig. 3.2 where feed enters 6$^{th}$ and 7$^{th}$ effects. It is assumed that feed is split between two effects equally. Live steam is fed to first two effects by steam splitting to fulfill heating requirements of the system. Three pre-heaters namely heater1, heater2 and heater3 are placed in between 4$^{th}$ and 10$^{th}$ effect in sequence as shown in the Fig. 3.2. Product is exiting from the second effect. The operating parameters for this system are shown in Table 3.2.

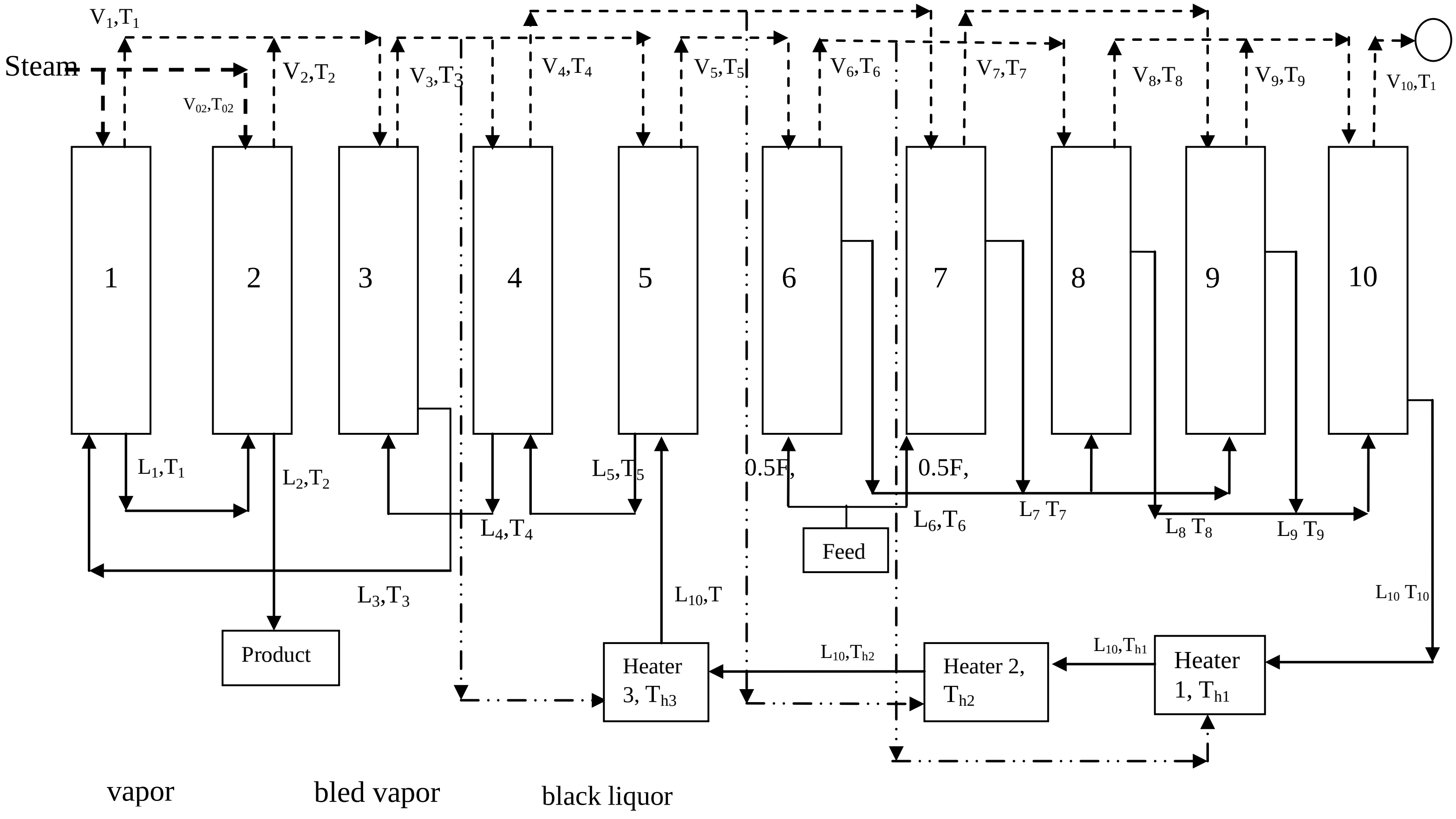



Fig. 3.2 Schematic diagram of ten effect evaporator system

Table 3.2 Operating parameters of ten effects evaporator system

| S. No | Parameter(s) | Value(s) |
|---|---|---|
| 1 | Total number of effects | 10 |
| 2 | Number of effects being supplied live steam | 2 |
| 3 | Live steam temperature in first and second effects | 140 °C |
| 4 | Black liquor inlet concentration | 0.16 |
| 5 | Black liquor final concentration | 0.68 |
| 6 | Liquor inlet temperature | 80$^{o}$C |
| 7 | Black liquor feed flow rate | 169800 kg/h |
| 8 | Last effect vapor temperature | 57.6 °C |
| 9 | Feed flow sequence | Mixed |

# CHAPTER 4

## MODEL DEVELOPMENT

In this Chapter, models for seven effect evaporator system, shown in Fig. 3.1 used for concentrating black liquor, are developed. The system selected for the present investigation can accommodate a variety of energy reduction schemes such as feed-, product- and condensate-flashing, steam splitting as well as liquor preheating with the help of bled vapor streams using pre-heaters. Along with this the present study accounts fouling conditions over heating surface and thus, a correlation of fouling factor is developed based on experimental data of fouling condition of black liquor given in the literature. As most of the models use temperature dependent physico-thermal properties of liquor/fluids it processes, for the present investigation, a number of correlations for the prediction of physico-thermal properties of black liquor and condensate are developed.

### 4.1. DEVELOPMENT OF CORRELATIONS FOR HEAT OF VAPORIZATION AND ENTHALPY

As steam/vapor enters different effects at different temperature properties of steam/vapor and condensate also vary with temperature. Thus, temperature dependent expressions of heat of vaporization and enthalpy are required to be developed. For this purpose data of heat of vaporization, enthalpy of condensate and enthalpy of vapor over the temperature range of 20-150°C, obtained from steam table, are plotted. A second order polynomial and linear trends are fitted on heat of vaporization and enthalpy curve as shown in Fig. 4.1 and 4.2, respectively. The developed expressions of heat of vaporization and enthalpy of condensate are shown through Eq. 4.1, 4.2 and 4.3, respectively.

$$\lambda = -0.0028T^2 - 2.1207T + 2496.1 \tag{4.1}$$

$$h = 4.2083T - 1.1734 \tag{4.2}$$

$$H = -0.0023T^2 + 2.0109T + 2497.9 \tag{4.3}$$

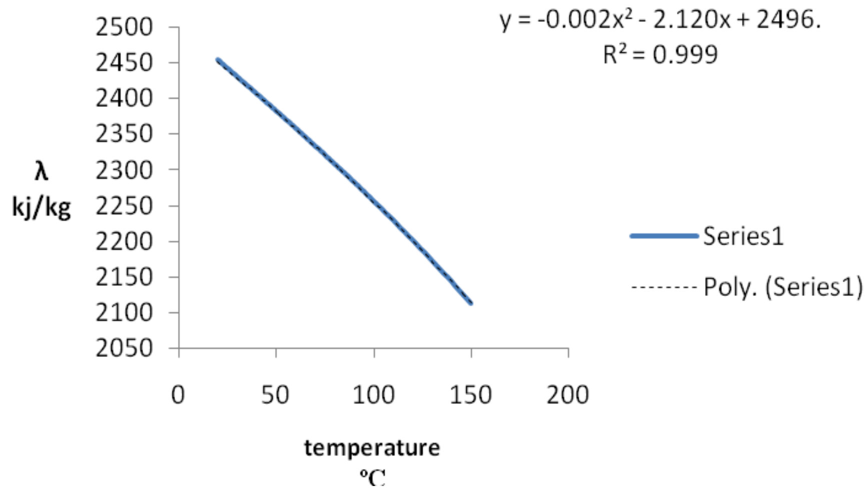



Fig. 4.1 Correlation of heat of vaporisation

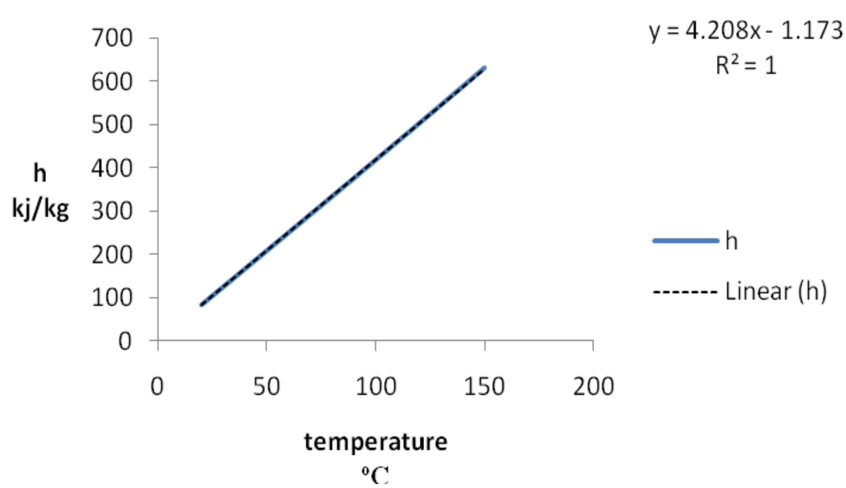



Fig. 4.2 Correlation of enthalpy of condensate

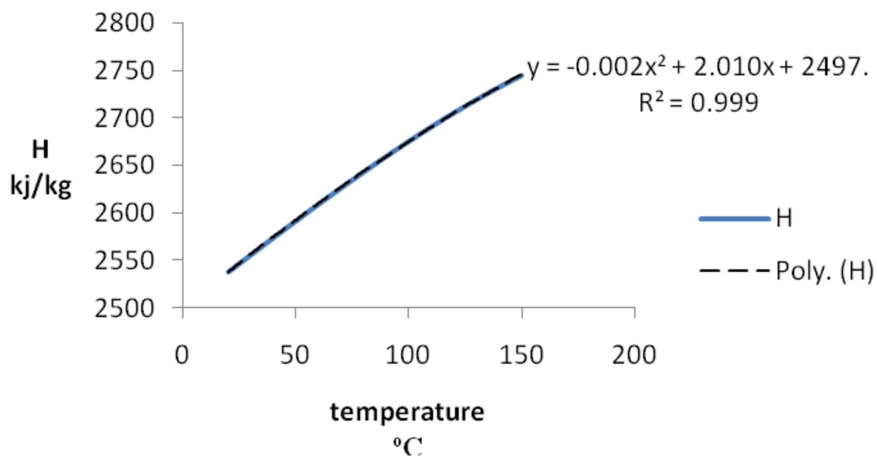



Fig 4.3 Correlation for enthalpy of vapor

## 4.2. CORRELATION FOR ENTHALPY OF BLACK LIQUOR

The expression for enthalpy of black liquor proposed in the work of Bhargava et al. (2008) and shown in Eq. 4.4 is used in the present work:

$$h_L = C_{PL} \ (T_L - C_5) \tag{4.4}$$

Where, $C_{PP} = C_1 \ (1 - C_{4X})$ (4.5)

Here values of coefficients $C_1$, $C_4$ and $C_5$ are 4187, 0.54 and 273, respectively.

## 4.3 MODEL FOR FOULING RESISTANCE AND OVERALL HEAT TRANSFER COEFFICIENT

Fouling affects the heat transfer rate due to deposition of suspended particle on the heat transfer surface and thus, reduces the overall heat transfer coefficient (U) considerably. One of the commonly practiced methods was to allow extra surface area to compensate for the heat loss suffered due to fouling film thickness, but then it led to the problem involving large area heat exchangers and difficulty in maintaining operability conditions.

To develop the correlation of rate of fouling it is required to study the parameters which affect rate of fouling. For this purpose the experimental study of (Muller-Steinhagen and Branch, 1997) is considered. They studied the effect of velocity as well as surface temperature on rate of fouling when evaporation of black liquor was carried out.

For a constant bulk temperature the rate of fouling increases with decrease in velocity. This effect is studied by Muller-Steinhagen and Branch (1997) and shown in Fig. 4.4 (Muller-Steinhagen and Branch, 1997). For convective heat transfer an increase in velocity increases the heat transfer coefficient, which reduces the overall rate of fouling. The effect of bulk temperature on the fouling rates is shown in Fig. 4.5 (Muller-Steinhagen and Branch, 1997). It is observed that the fouling rates increases with decreasing bulk temperature due to decrease in solubility of the scale-forming components.

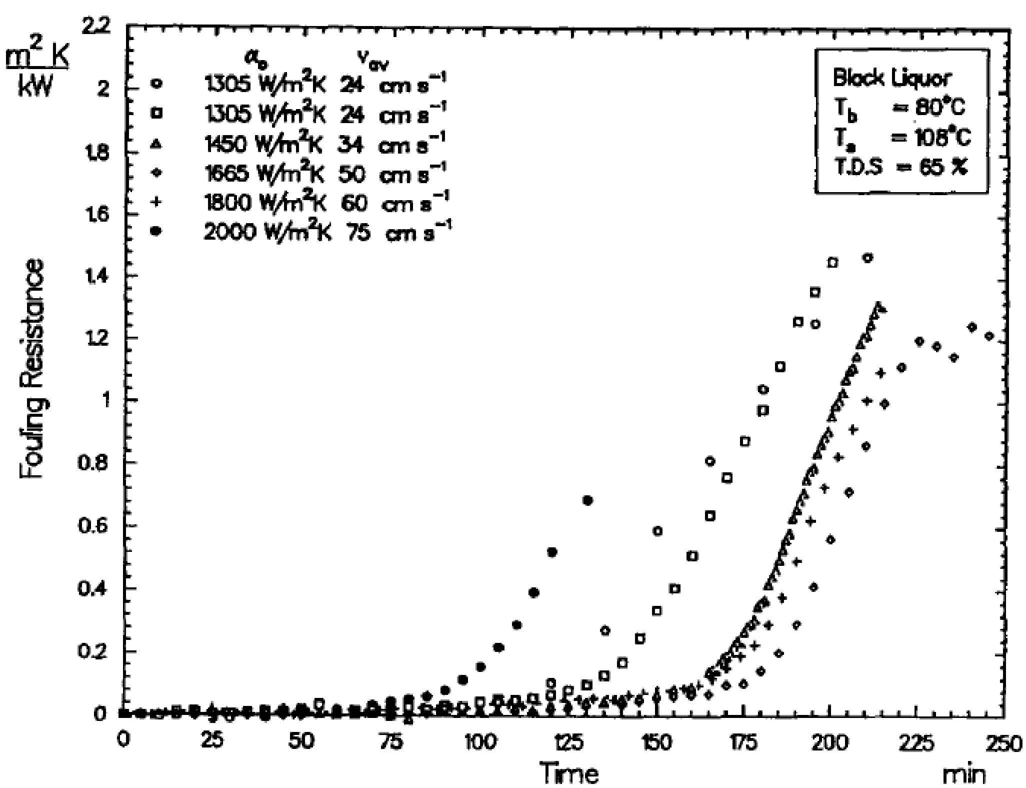



Fig. 4.4 Effect of velocity on fouling resistance

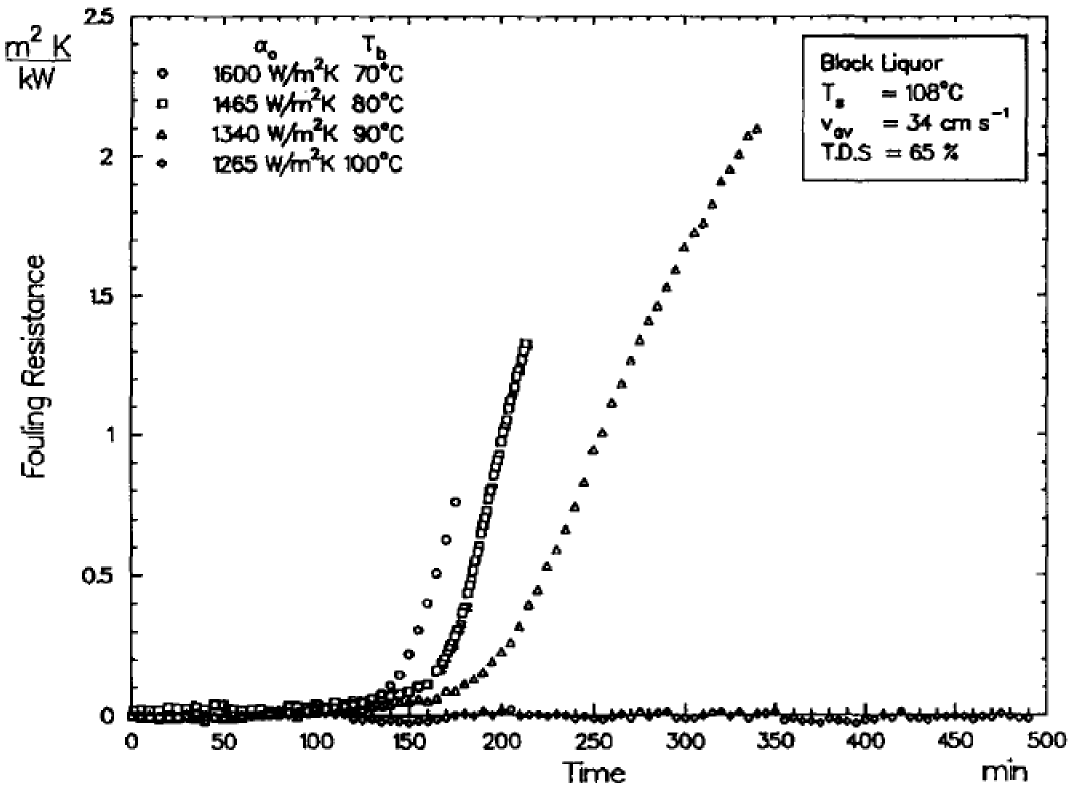



Fig. 4.5 Effect of bulk temperature on fouling resistance

The effects of velocity as well as bulk temperature on fouling resistance are shown in Figs. 4.4 and 4.5, respectively. The data required for developing the model of fouling resistance are extracted from these figures at different time domains such as over a single time domain the fouling resistances at various velocities are taken. Similar data sets of fouling resistances at various velocities over three time domains are extracted. Then average of these fouling resistances at individual velocities (four velocities) is calculated. To make the range of velocity more precise an average of consecutive velocities and corresponding fouling resistances are computed. It results in three velocities and their corresponding fouling resistances. A graph is plotted between the average fouling resistance, $R_{avg}$, and average velocity, $V_{avg}$, to observe and find the relation between the two. The trend followed by this data is found by fitting a second order polynomial line with $R^2$ as 1. A similar procedure is carried out to find the relation between temperature and fouling resistance. However, Fig. 4.5 shows the data of fouling resistance versus time for various bulk temperatures. So, to obtain the relation between $\Delta T$ and fouling resistance bulk temperature is subtracted from $T_s$ which is the deposit-fluid interface temperature. A graph is plotted between $\Delta T$ and fouling resistance and the equation best fitting the data is found. It is logarithmic expression with $R^2$ as 0.943. These plots are drawn to observe the variation of $R_{avg}$ with $\Delta T$ and $V_{avg}$.

Finally, two sets of data, one of $R_{avg}$ verses $V_{avg}$ and other of $R_{avg}$ verses $\Delta T$ are found. To study the variation of $\Delta T$ as well as velocity with fouling resistance the values of $\Delta T$ and velocity at common points of fouling resistance are calculated. It is done as: at the fouling resistance corresponding to the velocity is used to calculate the temperature at the same resistance using the relation between fouling resistance versus $\Delta T$. The computed values of $R_{avg}$, $V_{avg}$ and $\Delta T$ are summarized is Table 4.1.

Table 4.1 Computed values of $R_{avg}$, $V_{avg}$ and $\Delta T$

| $R_{avg}$ ($m^{2o}C/kW$) | $V_{avg}$ (cm/s) | $\Delta T$(◦C) |
|---|---|---|
| 0.7137 | 29 | 51.9 |
| 0.3238 | 42 | 25.07 |
| 0.2686 | 55 | 22.61 |

Regression analysis tool in Microsoft Excel is used to predict the relationship between data shown in Table 4.1. This tool is used to perform linear regression and found the variation of a dependent variable by the effect of one or more independent variable. Here it is used to find the effect of velocity and temperature difference on the fouling resistance. To use the regression analysis tool in Excel, the following path is used:

Data →data analysis→ regression→ input x and y range

By using this tool an equation is obtained that relates the fouling resistance with velocity and temperature difference. Hence, the final equation showing the relation between the three variables, presented in Table 4.1, is

$$R_{avg} = 0.0485 + 0.0137\Delta T - 0.00164\ V_{avg} \tag{4.6}$$

The correlation developed in Eq. 4.6 is used to account fouling resistance on the heat transfer surface of an evaporator. This fouling resistance is added to $U_c$ at clean condition to predict $U_d$ at fouled condition. The mathematical model of $U_c$ of different effects is developed by Bhargava (2004) based on plant data of seven effect evaporator system. This model is shown in Eq. 4.7. The values of coefficients used in Eq. 4.7 are presented in Table 4.2.

$$\frac{U_c}{2000} = a\left(\frac{\Delta T}{40}\right)^b \left(\frac{x_{avg}}{0.6}\right)^c \left(\frac{F_{avg}}{25}\right)^d \tag{4.7}$$

Table 4.2 Value of Coefficients of Eq. 4.6

| Effect No. | a | B | c | D | Maximum Percent Error Band |
|---|---|---|---|---|---|
| 1 and 2 | 0.0604 | -0.3717 | -1.2273 | 0.0748 | -11.32 to 7.25 |
| 3 to 7 | 0.1396 | -0.7949 | 0.0 | 0.1673 | -11.75 to 8.20 |

Thus, value of $U_d$ is predicted using Eq. 4.8.

$$\frac{1}{U_d} = R_{avg} + \frac{1}{U_c} \tag{4.8}$$

## 4.4 DEVELOPMENT OF MODEL FOR MEE SYSTEM

In the present section mathematical model is developed for seven effect evaporator system shown in Fig. 3.1. The model is developed in different stages. The first stage considers the simple system. Further, the model is improved to include variation in physical properties, BPR, feed-, product- and condensate- flashing, steam splitting and vapor bleeding from effects to use it in liquor preheating. These models are developed to show the effect of different energy reduction schemes as well as variations of actual MEE system.

### 4.4.1 Simple model for seven effect evaporator system

The schematic diagram of seven effect system selected for the development of simple model is presented in Fig. 4.6. In this system, live steam of amount, $V_0$, enters the steam chest of first effect at temperature $T_0$ and exits it as a condensate. The vapor generated in the first effect, as a result of evaporation of water, is moved to the vapor chest of the second effect and so on. It should be noted that effect numbers are assigned in increasing order in the direction of the movement of vapor stream. The vapor of last effect moves to a vacuum pump or a steam ejector or a barometric condenser. As a consequence of it, the first effect operates at the highest pressure (or highest temperature) whereas; last effect operates at lowest absolute pressure (or lowest temperature). In Fig. 4.6, feed follows the backward sequence i.e. it first enters into the seventh effect and then moves to sixth, then to fifth, then to fourth and so on. This process continues till the liquor reaches to first effect from where it comes out as product. Schematically, this can be represented by notation “Feed $\rightarrow$7$\rightarrow$6$\rightarrow$5$\rightarrow$4$\rightarrow$3$\rightarrow$2$\rightarrow$1$\rightarrow$ product”.

To develop simple model, Model-1, of the system shown in Fig. 4.6 following assumptions are made:

1. Vapor leaving an effect is at saturation condition.
2. Boiling point elevation is zero.

3. Variations in physical properties are negligible.

4. Heat losses from all effects are negligible.

5. Fouling resistance is negligible.

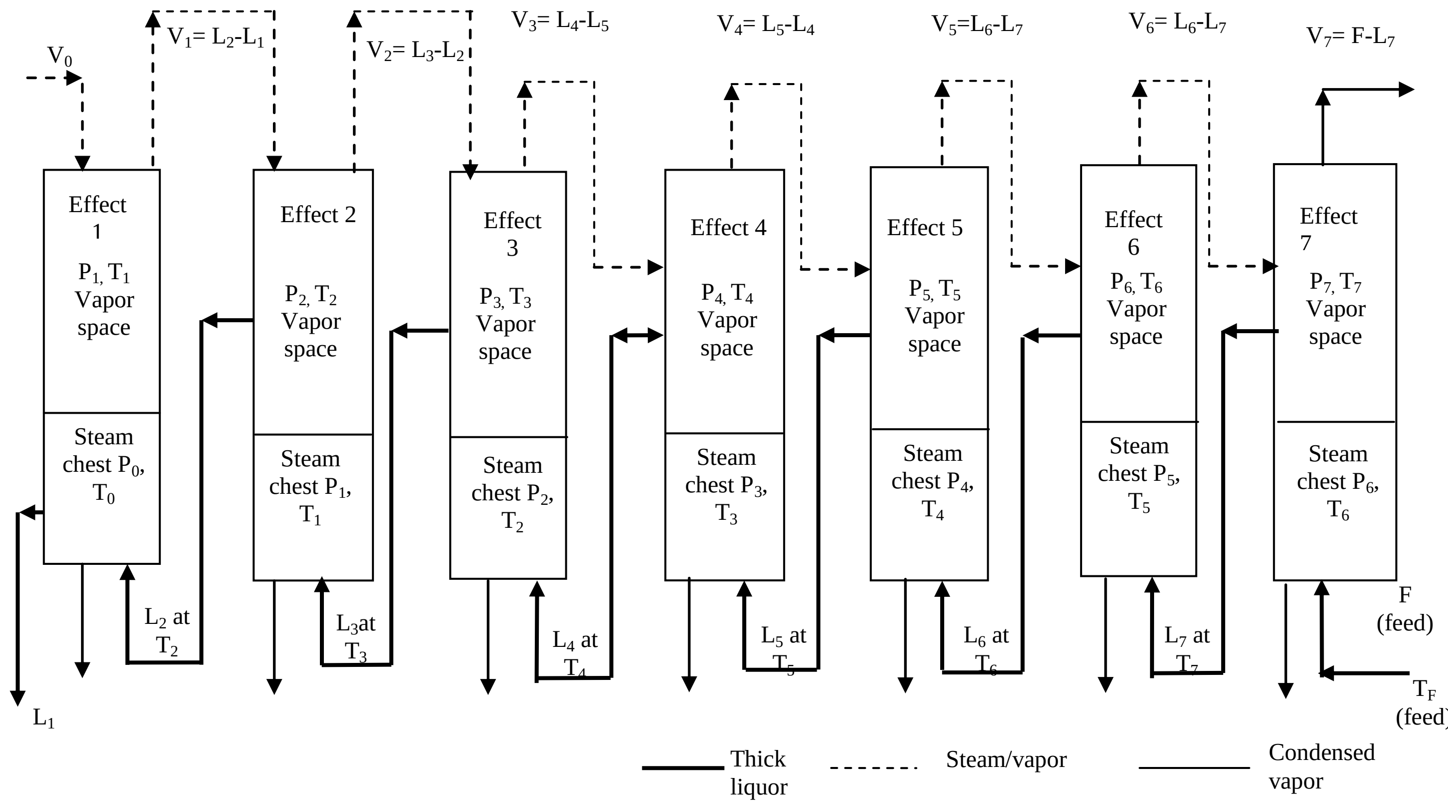



Fig. 4.6 Seven effect evaporator system with back ward feed

For seven effect evaporator system with backward sequence feed enters the last effect that is $7^{th}$ effect and heating medium that is steam enters the first effect. To develop Model-1 material and energy balance around first effect is derived as follows:

Energy balance around first effect

[liquor entering the effect from 2nd effect with sensible heat] + [steam entering the vapor chest with latent heat] = [vapor leaving the effect with latent heat] + [liquor leaving the effect with sensible heat]

$$[L_2C_pT_2] + [V_0\lambda_0] = [V_1(h_1 + \lambda_1)] + [L_1C_pT_1]$$

Or $[L_2C_pT_2] + [V_0\lambda_0] = [V_1\ h_1] + [V_1\lambda_1] + [L_1C_pT_1]$ (4.9)

As $V_1 = L_2 - L_1$ Eq. 4.9 becomes

$$[L_2C_pT_2] + [V_0\lambda_0] = [(L_2 - L_1)\ h_1] + [(L_2 - L_1)\ \lambda_1] + [L_1C_pT_1] \tag{4.10}$$

Enthalpy, $h_1 = C_p\ T_1$ is putting in Eq. 4.10 and then simplify to

$$[L_2C_pT_2] + [V_0\lambda_0] = [(L_2C_pT_1) - (L_1C_pT_1)] + [(L_2 - L_1)\lambda_1] + [L_1C_pT_1] \tag{4.11}$$

Eliminating and reducing the terms of Eq. 4.11 to get:

$$[L_2C_pT_2] + [V_0\lambda_0] = [L_2C_pT_1] + [(L_2 - L_1)\lambda_1] \tag{4.12}$$

Eq. 4.12 is rearranged to

$$[L_2C_p(T_2 - T_1)] + [V_0\lambda_0] - [(L_2 - L_1)\lambda_1] = 0 \tag{4.13}$$

Hence the equation for the first effect is:

$$f_1 = [L_2C_p(T_2 - T_1)] + [V_0\lambda_0] - [(L_2 - L_1)\lambda_1] \tag{4.14}$$

Further,

Heat transferred to the effect = latent heat supplied by the steam

$$U_1A_1(T_0 - T_1) = V_0\lambda_0$$

Or $f_2 = U_1A_1(T_0 - T_1) - (V_0\lambda_0)$ (4.15)

Similarly, for the next effects equations are derived, which are shown below:

2nd effect

$$f_3 = [L_3C_p(T_3 - T_2)] + [(L_2 - L_1)\lambda_1] - [(L_3 - L_2)\lambda_2] \tag{4.16}$$

$$f_4 = U_2A_2(T_1 - T_2) - (L_2 - L_1)\lambda_1 \tag{4.17}$$

3rd effect

$$f_5 = [L_4C_p(T_4 - T_3)] + [(L_3 - L_2)\lambda_2] - [(L_4 - L_3)\lambda_3] \tag{4.18}$$

$$f_6 = U_3A_3(T_2 - T_3) - (L_3 - L_2)\lambda_2 \tag{4.19}$$

4th effect

$$f_7 = [L_5C_p(T_5 - T_4)] + [(L_4 - L_3)\lambda_3] - [(L_5 - L_4)\lambda_4] \tag{4.20}$$

$$f_8 = U_4A_4(T_3 - T_4) - (L_4 - L_3)\lambda_3 \tag{4.21}$$

5th effect

$$f_9 = [L_6C_p(T_6 - T_5)] + [(L_5 - L_4)\lambda_4] - [(L_6 - L_5)\lambda_5] \tag{4.22}$$

$$f_{10} = U_5A_5(T_4 - T_5) - (L_5 - L_4)\lambda_4 \tag{4.23}$$

6$^{th}$ effect

$$f_{11} = [L_7C_p(T_7 - T_6)] + [(L_6 - L_5)\lambda_5] - [(L_7 - L_6)\lambda_6] \tag{4.24}$$

$$f_{12} = U_6A_6(T_5 - T_6) - (L_6 - L_5)\lambda_5 \tag{4.25}$$

7$^{th}$ effect

$$f_{13} = [FC_p(T_f - T_7)] + [(L_7 - L_6)\lambda_6] - [(F - L_7)\lambda_7] \tag{4.26}$$

$$f_{14} = U_7A_7(T_6 - T_7) - (L_7 - L_6)\lambda_6 \tag{4.27}$$

In above equations values of $C_p$ are taken from Eq. 4.5 where x is average of mass fraction of solute entering and leaving the first and last effect. The values of $\lambda_0$, $\lambda_1$, $\lambda_2$, $\lambda_3$… $\lambda_7$ are taken at respective temperatures of $T_0$, $T_1$, $T_2$, $T_3$… $T_7$ from the steam tables. These temperatures as well as U of different effects are taken from the work of Bhargava et al. (2008).

The simple model, Model-1, for seven effect evaporator system is developed through material and energy balance around each effect as shown through Eqs. 4.14 to 4.27. It is initial and basic step of model development where any variation or complication is not included.

In the similar lines model with fouling condition is developed where assumption that fouling resistance is negligible is relaxed. This model is referred as Model-2. Equations for this model are similar to that for Model-1 except that values of U are computed as follows: For predicting values of $U_d$ the effect of fouling is included to U of Model-1 and resulting values are used for deriving the equations. For this purpose, $R_{avg}$ which is derived from Eq. 4.6 for each effect with the data obtained from the work of Bhargava et al. (2008). Then based on Eq. 4.8 $U_d$ is calculated for each effect which is used in place of U in Eqs. 4.14- 4.27.

**4.4.2 Model with steam splitting**

Steam that is fed to the first effect in Model-1 is split among first and second effect. The vapor generated from first and second effects are combined together to enter into third effect. However, the vapor produced in third effect is used as heating medium in fourth effect and vapor of fourth effect is utilized in fifth effect and so on.

The schematic diagram of the system is similar to Fig. 4.6 except that in first two effects steam enters as $V_{01}$ and $V_{02}$. To include steam splitting Model-3 is developed. In this model it is assumed that 0.5 fraction of total steam enters in first effect and remaining amount enters in second effect ($V_{01}$=$V_{02}$=0.5$V_0$). The data used for this model such as Cp, $\lambda$, U and A remains same as that used for the Model-1. The assumptions for Model-3 are as follows: vapor leaving an effect is at saturation condition. BPR, variations in physical properties and heat loss from effects are negligible. In Model-3 equations for first effect are derived using mass and energy balance which are shown below:

For Model-3 as steam is entering to first and second effect vapor to these effects is replaced by half of total steam i.e., $0.5V_0$. The equations for these effects are shown below:

1st effect

$$f_1 = [L_2C_p(T_2 - T_1)] + 0.5[V_0\lambda_{01}] - [(L_2 - L_1)\lambda_1] \tag{4.28}$$

$$f_2 = U_1A_1(T_0 - T_1) - 0.5(V_0\lambda_{01}) \tag{4.29}$$

2nd effect

$$f_3 = [L_3C_p(T_3 - T_2)] + [0.5[V_0\lambda_{02}]] - [(L_3 - L_2)\lambda_2] \tag{4.30}$$

$$f_4 = U_2A_2(T_1 - T_2) - 0.5(V_0\lambda_{02}) \tag{4.31}$$

The vapor streams coming out from first and second effect enter the third effect hence; the equations for third effect are modified as:

$$f_5 = [L_4C_p(T_4 - T_3)] + [(V_1 + V_2)\lambda_2] - [(L_4 - L_3)\lambda_3]$$

$$= [L_4C_p(T_4 - T_3)] + [(L_2 - L_1)\lambda_1] + [(L_3 - L_2)\lambda_2] - [(L_4 - L_3)\lambda_3] \tag{4.32}$$

$$f_6 = U_3A_3(T_2 - T_3) - [(L_2 - L_1)\lambda_1] + [(L_3 - L_2)\lambda_2] \tag{4.33}$$

Equations for fourth to seventh effects will be same as Eqs. 4.20 to 4.27. Thus, Model-3 which includes steam splitting involves total 14 equations.

The model with steam splitting and fouling condition is developed and referred as Model-4. The equation for this model is similar to that for Model-3 except that values of U are computed using Eq. 4.8.

### 4.4.3 Model with variation in physical properties and BPR

The actual MEE system cannot be simulated without considering variation in physical properties. These properties are specific heat capacity of liquor, $C_p$, latent heat of vaporization, $\lambda$, and BPR.

To consider variations in these properties Eq. 4.1 and 4.4 are used for λ and $C_p$, respectively. The variation in BPR can be considered using following expressions (Bharagav et al., 2008):

$$\tau = 20\times (0.1 + x)^2 \tag{4.34}$$

Using all variations of physical properties model of seven effect evaporator system is developed and named as Model-5. The assumptions for Model-5 are as follows: vapor leaving an effect is at saturation condition and heat loss from effect is negligible. Based on material and energy balances equations for first to seventh effects are formulated and shown below:

1st effect

$$f_1 = [L_2 C_{p2}(T_2 + \tau_2)] + [0.5V_0\lambda_{01}] - [L_1 C_{p1}(T_1 + \tau_1)] - [(L_2 - L_1)(\lambda_1 + (4.2T_1))] \tag{4.35}$$

$$f_2 = U_1 A_1 (T_{01} - T_1 - \tau_1) - (0.5V_0\lambda_{01}) \tag{4.36}$$

2nd effect

$$f_3 = [L_3 C_{p3}(T_3 + \tau_3)] + [0.5V_0\lambda_{02}] - [L_2 C_{p2}(T_2 + \tau_2)] - [(L_3 - L_2)(\lambda_2 + (4.2T_2))] \tag{4.37}$$

$$f_4 = U_2 A_2 (T_{02} - T_2 - \tau_2) - (0.5V_0\lambda_{02}) \tag{4.38}$$

3rd effect

$$f_5 = [L_4 C_{p4}(T_4 + \tau_4)] - [L_3 C_{p3}(T_3 + \tau_3)] + (L_2 - L_1)\lambda_1 + (L_3 - L_2)\lambda_2 - [(L_4 - L_3)(\lambda_3 + (4.2T_3))] \tag{4.39}$$

$$f_6 = U_3 A_3 \left(\frac{T_1 + T_2}{2} - T_3 - \tau_3\right) - (L_2 - L_1)\lambda_1 - (L_3 - L_2)\lambda_2 \tag{4.40}$$

4th effect

$$f_7 = [L_5 C_{p5}(T_5 + \tau_5)] - [L_4 C_{p4}(T_4 + \tau_4)] + (L_4 - L_3)\lambda_3 - [(L_5 - L_4)(\lambda_4 + (4.2T_4))] \tag{4.41}$$

$$f_8 = U_4 A_4 (T_3 - T_4 - \tau_4) - [(L_4 - L_3)\lambda_3] \tag{4.42}$$

5th effect

$$f_9 = [L_6 C_{p6}(T_6 + \tau_6)] - [L_5 C_{p5}(T_5 + \tau_5)] + (L_5 - L_4)\lambda_4 - [(L_6 - L_5)(\lambda_5 + (4.2T_5))] \quad (4.43)$$

$$f_{10} = U_5 A_5 (T_4 - T_5 - \tau_5) - [(L_5 - L_4)\lambda_4] \quad (4.44)$$

$6^{th}$ effect

$$f_{11} = [L_7 C_{p7}(T_7 + \tau_7)] - [L_6 C_{p6}(T_6 + \tau_6)] + (L_6 - L_5)\lambda_5 - [(L_7 - L_6)(\lambda_6 + (4.2T_6))] \quad (4.45)$$

$$f_{12} = U_6 A_6 (T_5 - T_6 - \tau_6) - [(L_6 - L_5)\lambda_5] \quad (4.46)$$

$7^{th}$ effect

$$f_{13} = [F C_{pf} T_f] - [L_7 C_{p7}(T_7 + \tau_7)] + (F - L_7)\lambda_7 - [(L_7 - L_6)\lambda_6] \quad (4.47)$$

$$f_{14} = U_7 A_7 (T_6 - T_7 - \tau_7) - [(L_7 - L_6)\lambda_6] \quad (4.48)$$

The variations in physical properties are accounted as: considering equal $\Delta T$ and equal vaporization in each effect temperatures as well as concentrations of each effect are found. Using these parameters the values of $\lambda$, $C_p$ and $\tau$ are found. These values are used in Eqs. 4.35 to 4.48 and then solved.

The model with steam splitting, variation in physical properties and fouling condition are developed and referred as Model-6 where equations are similar to that for Model-5 except that values of U are computed using Eq. 4.8 instead of Eq. 4.7.

### 4.4.4 Model with condensate flashing

In this section along with steam splitting and variation in physical properties condensate flashing is also included in the model and called it as Model-7. The condensate (water in present case), which exits from steam/vapor chest of an effect, contains sufficient amount of sensible heat which can be put to use. This sensible heat can be extracted by means of flashing which will produce low pressure vapor. This vapor can be used as a heating medium in vapor chests of appropriate effects and thereby can improve steam economy of the whole system. The schematic diagram of seven effect evaporator system with provisions of condensate flashing is shown in Fig. 4.7 and is used for the model development.

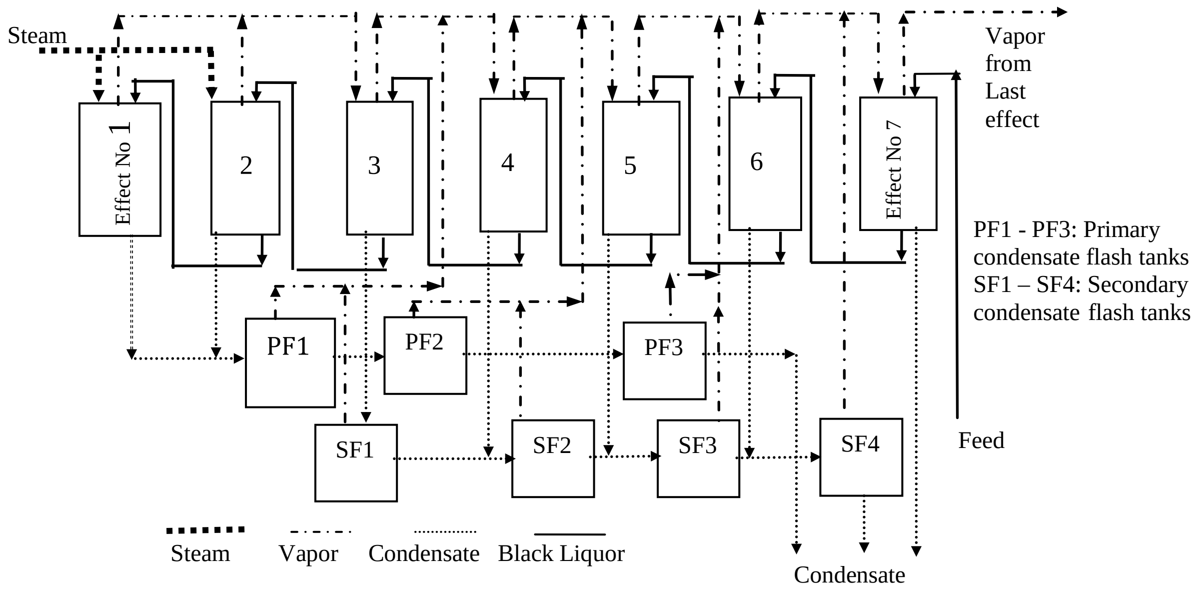



Fig. 4.7 Seven effect evaporator system with condensate flashing

Condensate coming out from the steam chest of an effect enters into a flash tank where it is ‘flashed’. Vapor generated through flashing is combined with other vapor streams coming out from other flash tanks and mixed vapor stream is utilized as a heating medium in subsequent effect. The amount of vapor generated through flashing can be computed based on material and energy balance around a flash tank shown in Fig 4.8. Here condensate of amount $V_0$ is entering to first primary flash tank, PF1, at $T_0$. $V_{1v}$ is the amount of vapor leaving the flash tank at temperature $T_3$ and $V_{11}$ is the condensate leaving the flash tank that is used in second flash tank, PF2, for being flashed at $T_3$. The expression of $V_{1v}$ can be derived as given below:

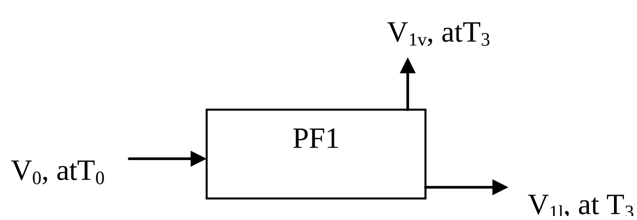



Fig. 4.8 Schematic diagram of primary condensate flash tank

Material balance around PF1: $V_0 = V_{1v} + V_{1l}$ (4.49)

Energy balance around PF1: $V_0 h_0 = V_{1v} H_3 + V_{1l} h_3$ (4.50)

Solving Eq. 4.49 and 4.50,

$$V_{1v} = V_0 \frac{(h_0 - h_3)}{(H_3 - h_3)} \tag{4.51}$$

The values of $h_0$, $h_3$ and $H_3$ are computed at temperatures $T_0$ and $T_3$. Based on assumption of equal temperature difference, $T_0$ and $T_3$ are found as 140°C and 110°C, respectively. At these values of temperatures $V_{1v}$ is predicted as

$$V_{1v} = 0.04193 V_0 \tag{4.52}$$

The assumptions made for Model-7 include vapor leaving an effect is at saturation condition and heat losses from all effects are negligible. As vapor generated through flashing is entering into vapor chest of fourth to seventh effects the governing equation for these effect will be modified. However, for first to third effects the equations are similar to Eq. 4.35 to 4.40 of Model-5. The equation for fourth effect is derived based on mass and energy balance around the system, shown in Fig. 4.9 as given below:

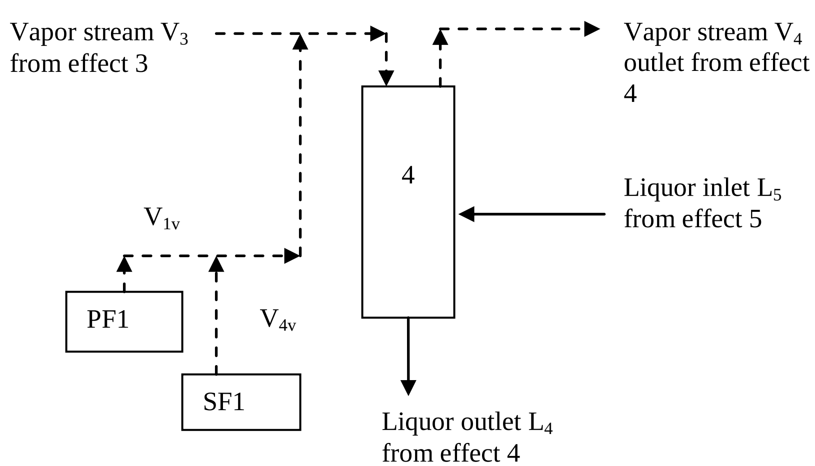



Fig 4.9 material and energy balance around 4th effect with flashing

Energy Balance around 4th effect gives:

[Sensible heat of liquor ($L_5$)]+ [latent heat of vapor ($V_3$)]+ [latent heat of vapor streams from PF1 and SF1 ($V_{1v}$+$V_{4v}$)] = [Sensible heat of liquor ($L_4$)] + [heat of vapor stream ($V_4$)]

$$\left[\left(L_5C_{p4}(T_5+\tau_5)\right)\right]+[(L_4-L_3)\lambda_3]+[(V_{1v}+V_{4v})\lambda_3] = \left[\left(L_4C_{p4}(T_4+\tau_4)\right)\right]+[(L_5-L_4)(\lambda_4+(4.2T_4))] \tag{4.53}$$

Substituting the value of $V_{1v}$ from Eq. 4.52 to Eq. 4.53 and rearranging:

$$f_7 = [L_5C_{p5}(T_5+\tau_5)]-[L_4C_{p4}(T_4+\tau_4)]-[(L_5-L_4)\{\lambda_4+(4.2T_4)\}]+[\{(L_4-L_3)+(0.04193V_0)+0.02832(L_2-L_1+L_3-L_2)\}\lambda_3] \tag{4.54}$$

In Eq. 4.54 the value 0.02832 is predicted using temperatures related to flash tank SF1 in the similar manner as Eq. 4.52.

Energy balance at the steam side of 4th effect gives:

$$f_8 = [U_4A_4(T_3-T_4-\tau_4)]-[\{(L_4-L_3)+(0.04193V_0)+0.02832(L_3-L_1)\}\lambda_3] \tag{4.55}$$

Likewise the flashed vapors coming out from each flash tank are added to the vapor stream going into the respective effects are included in the equations as follows:

5th effect

$$f_9 = [L_6C_{p6}(T_6+\tau_6)]-[L_5C_{p5}(T_5+\tau_5)]-[(L_6-L_5)\{\lambda_5+(4.2T_5)\}]+[\{(L_5-L_4)+(0.02198V_0)+0.0229(L_4-L_1+0.04193V_0)\}\lambda_4] \tag{4.56}$$

$$f_{10} = [U_5A_5(T_4-T_5-\tau_5)]-[\{(L_5-L_4)+(0.02198V_0)+0.0229(L_4-L_1+0.04193V_0)\}\lambda_4] \tag{4.57}$$

6th effect

$$f_{11} = [L_7C_{p7}(T_7+\tau_7)]-[L_6C_{p6}(T_6+\tau_6)]-[(L_7-L_6)\{\lambda_6+(4.2T_6)\}]+[\{(L_6-L_5)+(0.02177V_0)+0.02359(L_5-L_1+0.06392V_0)\}0.023\lambda_5] \tag{4.58}$$

$$f_{12} = [U_6A_6(T_5 - T_6 - \tau_6)] - [\{(L_6 - L_5) + (0.02177V_0) + 0.02359(L_5 - L_1 + 0.06392V_0)\}\lambda_6]$$

(4.59)

7th effect

$$f_{13} = (FC_{pf}T_f) - \{L_7C_{p7}(T_7 + \tau_7)\} - \{(F - L_7)\lambda_7\} + [\{(L_7 - L_6) + 0.05134(L_6 - L_1 + 0.08569V_0)\}\lambda_6]$$

(4.60)

$$f_{14} = [U_7A_7(T_6 - T_7 - \tau_7)] - [\{(L_7 - L_6) + 0.05134(L_6 - L_1 + 0.08569V_0)\}\lambda_6] \quad (4.61)$$

For Model-7 the variations in physical properties are accounted in the similar manner as described under Section 4.4.3.

The model with steam splitting, variation in physical properties, condensate flashing and fouling condition are developed and referred as Model-8 where equations are similar to that for Model-7 except that values of U are computed using Eq. 4.8.

### 4.4.5 Model with feed and product flashing

The model (Model-7), developed in Section 4.4.4, is further extended in the present section to incorporate provisions of feed and product flashing in the seven effect evaporator system. These provisions are used in the system for two purposes: first, it helps water to be evaporated from feed and product without using steam/vapor and second, vapor generated from flashing of feed and product, is used as a heating medium at appropriate effects. Thus, these provisions enhance the steam economy of the system. Fig. 3.1 shows the schematic diagram of this system in which FFT (feed flash tank) and PFT (product flash tank) are included for feed and product flashing, respectively.

To obtain values of product and feed flash flow rates, $L_i$ and $F_V$, generated through flashing in PFT and FFT, the following procedure is used (Bhargava et al., 2008):

$$a_1L_i^3 + a_2L_i^2 + a_3L_i + a_4 = 0 \quad (4.62)$$

Eq. 4.62 is used to obtain $L_i$ which is the black liquor outlet flow rate from $i^{th}$ flash tank. Where coefficients $a_1$, $a_2$, $a_3$ and $a_4$ of the cubic polynomial are functions of input liquor parameters as given below:

$$a_1 = H_{Vout} - C_1 T_{out} - C_1 C_2^2 C_3 + C_1 C_5 \tag{4.63}$$

$$a_2 = L_{in} h_{Lin} + L_{in} x_{in} (C_1 C_4 T_{out} - 2C_1 C_2 C_3 + C_1 C_2^2 C_3 C_4 - C_1 C_4 C_5) - L_{in} H_{Vout} \tag{4.64}$$

$$a_3 = (L_{in} x_{in})^2 (2C_1 C_2 C_3 C_4 - C_1 C_3) \tag{4.65}$$

$$a_3 = (L_{in} x_{in})^3 (C_1 C_3 C_4) \tag{4.66}$$

Where $C_1$, $C_2$, $C_3$, $C_4$ and $C_5$ are the constants with values 4187, 0.1, 20, 0.54 and 273 respectively.

The unknown parameters such as $H_{Vout}$, $h_{Lin}$, $L_{in}$ and $x_{in}$ are obtained from initial values of temperatures predicted assuming equal temperature difference and equal vaporization in each effect.

The coefficients thus obtained through Eq. 4.63 to 4.66 are in turn used to solve Eq. 4.62 and compute the value of $L_i$. The difference of $L_{in}$ and $L_i$ is the vapor produced through product and feed flashing. Thus values of $L_i$ and $F_V$ can be obtained.

The model developed with the induction of feed and product flashing in Model-7 is referred as Model-9 where steam splitting, variation in physical properties and condensate flashing are also accounted. For Model-9 it is assumed that vapor leaving an effect is at saturation condition and heat losses from all effects are negligible.

As vapor generated through product and feed flashing are entering into vapor chest of fourth and seventh effect, respectively, the governing equation for these effect will be modified. Thus, equations for $1^{st}$, $2^{nd}$, $3^{rd}$, $5^{th}$ and $6^{th}$ effects are similar to equations of these effects for Model-7.

The equations for $4^{th}$ and $7^{th}$ effects are derived based on mass and energy balance as shown for $4^{th}$ effect of Model-7 and given below:

$4^{th}$ effect

$$f_7 = [L_5 C_{p5}(T_5 + \tau_5)] - [L_4 C_{p4}(T_4 + \tau_4)] - [(L_5 - L_4)\{\lambda_4 + (4.2T_4)\}] + [\{(L_4 - L_3) + L_i + (0.04193V_0) + 0.02832(L_3 - L_1)\}\lambda_3] \quad (4.67)$$

$$f_8 = [U_4 A_4 (T_3 - T_4 - \tau_4)] - [\{(L_4 - L_3) + L_i + (0.04193V_0) + 0.02832(L_3 - L_1)\}\lambda_3] \quad (4.68)$$

For $7^{th}$ effect feed flashing is included as shown below:

$7^{th}$ effect

$$f_{13} = (FC_{pf}T_f) - \{L_7 C_{p7}(T_7 + \tau_7)\} - \{(F - L_7)\lambda_7\} + [\{(L_7 - L_6) + F_V + 0.05134(L_6 - L_1 + 0.08569V_0)\}\lambda_6] = 0 \quad (4.69)$$

$$f_{14} = [U_7 A_7 (T_6 - T_7 - \tau_7)] - [\{(L_7 - L_6) + F_V + 0.05134(L_6 - L_1 + 0.08569V_0)\}\lambda_6] \quad (4.70)$$

The model with steam splitting, variation in physical properties, feed, product and condensate flashing and fouling conditions are developed and referred as Model-10 where equations are similar to that of Model-9 except that values of U are computed using Eq. 4.8 instead of Eq. 4.7.

### 4.4.6 Model with vapor bleeding

Vapor bleeding is used to preheat the liquor that is coming out from an effect using vapor stream extracted from the vapor leaving from one of the previous effects. As shown in the Fig. 4.10 there are four preheaters placed in between $3^{rd}$ and $4^{th}$ effect and so on till $7^{th}$ effect. The vapor 'bled' or extracted from the vapor emerging out from $2^{nd}$ effect is used to preheat liquor that is coming from the $4^{th}$ effect (before it enters the $3^{rd}$ effect) using a preheater placed in between $3^{rd}$ and $4^{th}$ effect. Similarly, other preheaters are placed.

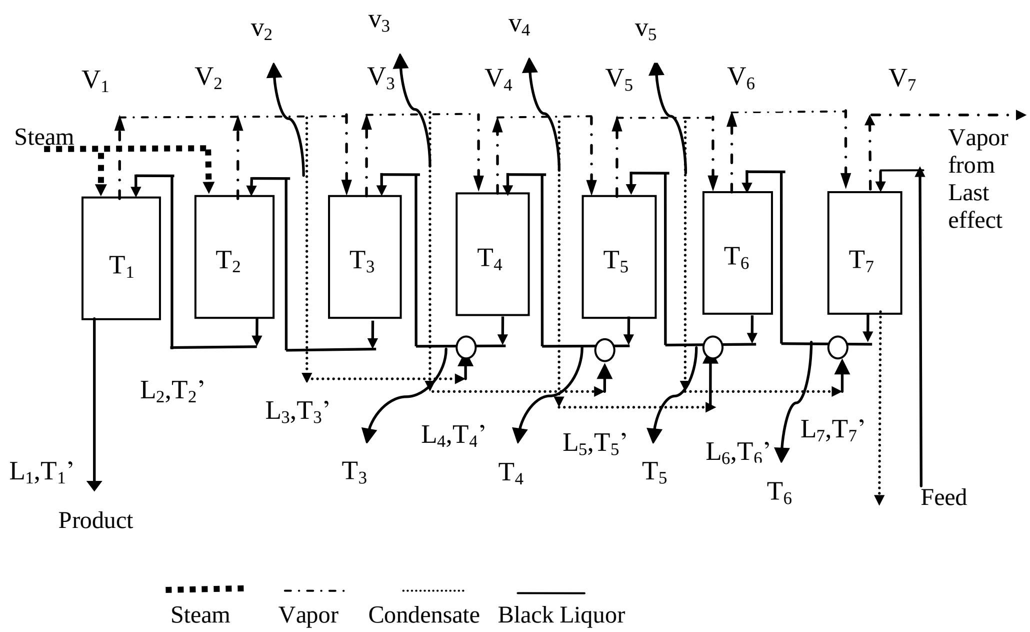



Fig 4.10 Schematic diagram of seven effect system with vapor bleeding

The material and energy balance equations are derived around each effect and additionally done for each preheater. This results in the set of equations for Model-11. For this model, equations for $1^{st}$, $2^{nd}$ and $7^{th}$ effect will be similar to that of Model-3. For rest of the effects equations are developed as described below. The schematic diagram of preheater-1, which is in between $3^{rd}$ and $4^{th}$ effect, is shown in Fig. 4.11:

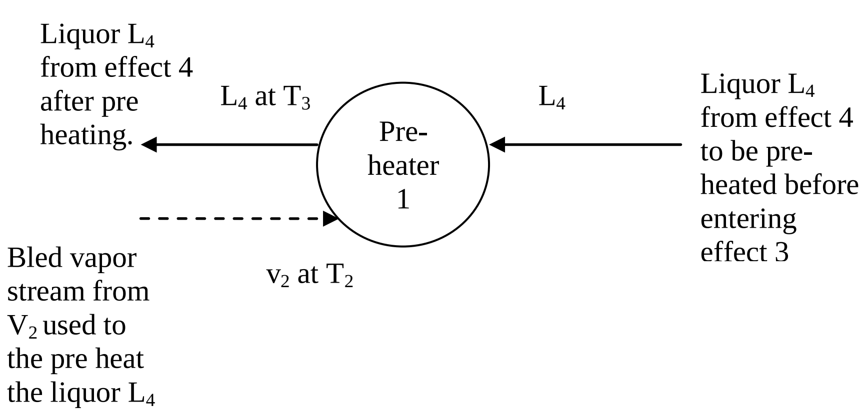



Fig 4.11 Schematic diagram of pre-heater 1

Material balance around preheater-1 is given as:

$$v_2\lambda_2 = L_4C_{p4}(T_3 - \tau_4) \tag{4.71}$$

Equations of 3rd effect can be developed using Fig. 4.12:

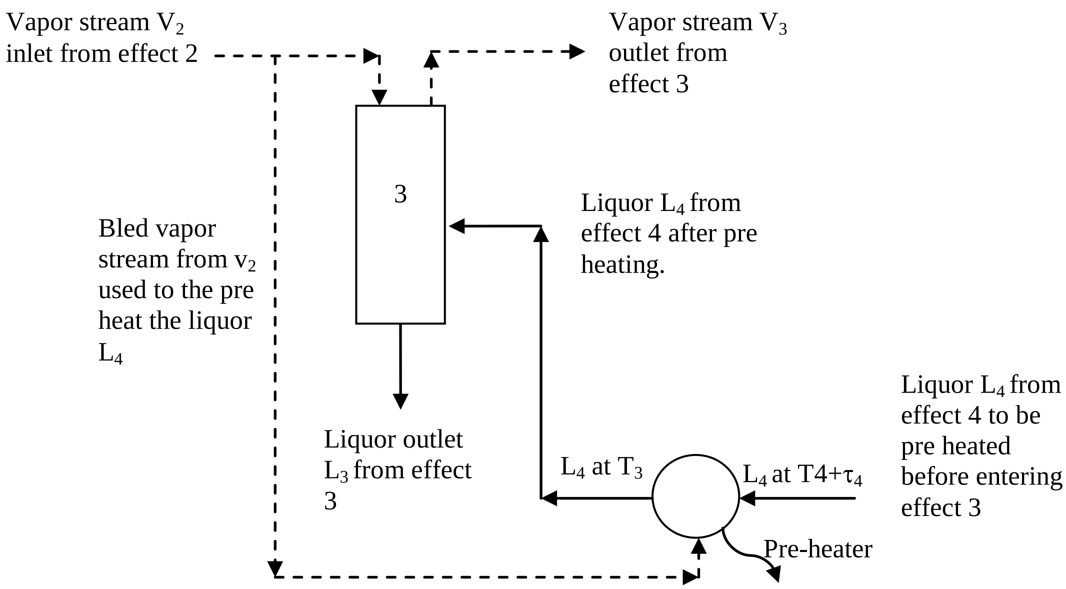



Fig 4.12 Schematic diagram of 4th effect with vapor bleeding

3rd effect

$$f_5 = L_4C_{p4}T_3 + (V_1 + V_2 - v_2)\lambda_2 - L_3C_{p3}\tau_3 - V_3(\lambda_3 + 4.2T_3) \tag{4.72}$$

$$f_6 = U_3A_3\left(\left(\frac{T_1+T_2}{2}\right) - \tau_3\right) - (V_1 + V_2 - v_2)\lambda_2 = 0 \tag{4.73}$$

$$f_7 = v_2\lambda_2 - L_4C_{p4}(T_3 - \tau_4) = 0 \tag{4.74}$$

Similarly, equations for 4th, 5th and 6th effects are modified with the inclusion of balance equations of preheaters as shown below:

4th effect

$$f_8 = L_5C_{p5}T_4 + (V_3 - v_3)\lambda_3 - L_4C_{p4}\tau_4 - V_4(\lambda_4 + 4.2T_4) = 0 \tag{4.75}$$

$$f_9 = U_4A_4(T_3 - \tau_4) - (V_3 - v_3)\lambda_3 = 0 \tag{4.76}$$

$$f_{10} = v_3\lambda_3 - L_5C_{p5}(T_4 - \tau_5) = 0 \tag{4.77}$$

5th effect

$$f_{11} = L_6C_{p6}T_5 + (V_4 - v_4)\lambda_4 - L_5C_{p5}\tau_5 - V_5(\lambda_5 + 4.2T_5) = 0 \tag{4.78}$$

$$f_{12} = U_5A_5(T_4 - \tau_5) - (V_4 - v_4)\lambda_4 = 0 \tag{4.79}$$

$$f_{13} = v_4\lambda_4 - L_6C_{p6}(T_5 - \tau_6) = 0 \tag{4.80}$$

6th effect

$$f_{14} = L_7C_{p7}T_6 + (V_5 - v_5)\lambda_5 - L_6C_{p6}\tau_6 - V_6(\lambda_6 + 4.2T_6) = 0 \tag{4.81}$$

$$f_{15} = U_6A_6(T_5 - \tau_6) - (V_5 - v_5)\lambda_5 = 0 \tag{4.82}$$

$$f_{16} = v_5\lambda_5 - L_7C_{p7}(T_6 - \tau_7) = 0 \tag{4.83}$$

The model with steam splitting, variation in physical properties and vapor bleeding under fouling condition are developed and referred as Model-12 where equations are similar to that for Model-11 except that values of U are computed using Eq. 4.8 instead of Eq. 4.7.

### 4.4.7 Model with vapor bleeding and flashing

This model is the summation of all the variations considered so far. It includes steam splitting, variation in physical properties, feed, product and condensate flashing and vapor bleeding. The portion of vapor that is entering the next effect is bled to preheat liquor entering the following effect and also its condensate is added up with other condensates to get flashed in the flash tank. The variation can be shown by performing material and energy balance around 4th effect which has a pre-heater to heat liquor coming out from it before it enters the 3rd effect and vapor streams which has the streams $V_{1v}$, $V_{4v}$, and $L_i$, combined with $V_3$ entering into the steam chest as shown in Fig. 4.13.

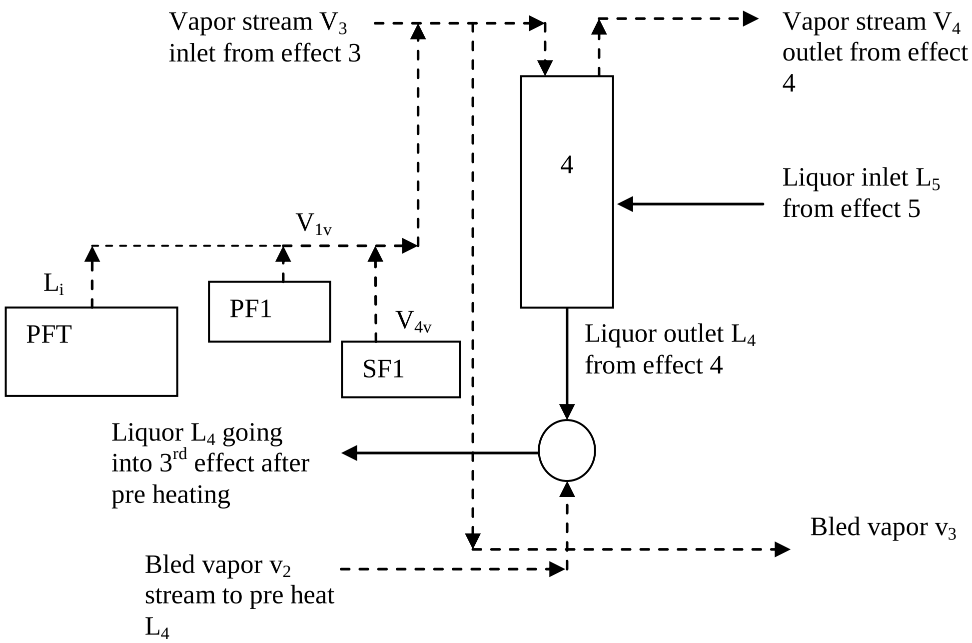



Fig 4.13 Schematic diagram of 4th effect with vapor bleeding and flashing

Energy balance around 4th effect is given as

[Sensible heat of liquor ($L_5$)]+ [latent heat of vapor ($V_3$)] – [Latent heat of vapor ($v_3$)] +

[latent heat of vapor streams from PFT, PF1 and SF1 ($L_i$+$V_{1v}$+$V_{4v}$)]

= [Sensible heat of liquor ($L_4$)] + [heat of vapor stream ($V_4$)]

$$\left[\left(L_5C_{p5}(T_5+\tau_5)\right)\right]+[\{(L_4-L_3)-v_3\}\lambda_3]+[(L_i+V_{1v}+V_{4v})\lambda_3]=\left[\left(L_4C_{p4}(T_4+\tau_4)\right)\right]+[(L_5-L_4)(\lambda_4+(4.2T_4))] \tag{4.84}$$

Rearranging Eq. 4.84:

$$\left[\left(L_5C_{p5}(T_4)\right)\right]-\left[\left(L_4C_{p4}(T_4+\tau_4)\right)\right]-[(L_5-L_4)(\lambda_4+(4.2T_4))]+[\{(L_4-L_3)+L_i-v_3+(0.0412859V_0)+0.023970(L_3-L_1)\}\lambda_3]=0 \tag{4.85}$$

Further,

$$[U_4A_4(T_3-(T_4+\tau_4))]-[\{(L_4-L_3-v_3)+L_i+(0.0412859V_0)+0.023970(L_3-L_1)\}\lambda_3]=0 \tag{4.86}$$

Material and energy balance around the pre-heater

$$v_3\lambda_3 = L_5C_{p5}\{T_4 - (T_5 + \tau_6)\} \tag{4.87}$$

Similarly for other effects the equations can be given as:

5th effect

$$\left[\left(L_6C_{p6}(T_5)\right)\right] - \left[\left(L_5C_{p5}(T_5 + \tau_5)\right)\right] - [(L_6 - L_5)(\lambda_5 + (4.2T_5))] + [\{(L_5 - L_4) - v_4 + (0.022631V_0) + 0.024357(L_4 - L_1 + 0.041286V_0)\}\lambda_4] = 0 \tag{4.88}$$

$$[U_5A_5(T_4 - (T_5 + \tau_5))] - [\{(L_5 - L_4) - v_4 + (0.022631V_0) + 0.024357(L_4 - L_1 + 0.041286V_0)\}\lambda_3] = 0 \tag{4.89}$$

$$v_4\lambda_4 - [L_6C_{p6}\{T_5 - (T_6 + \tau_6)\}] = 0 \tag{4.90}$$

6th effect

$$\left[\left(L_7C_{p7}(T_6)\right)\right] - \left[\left(L_6C_{p6}(T_6 + \tau_6)\right)\right] - [(L_7 - L_6)(\lambda_6 + (4.2T_6))] + [\{(L_6 - L_5) - v_5 + (0.0217727V_0) + 0.0232594(L_5 - L_1 + 0.06391V_0)\}\lambda_5] = 0 \tag{4.91}$$

$$[U_6A_6(T_5 - (T_6 + \tau_6))] - [\{(L_6 - L_5) - v_5 + (0.0217727V_0) + 0.023259(L_5 - L_1 + 0.063917V_0)\}\lambda_5] = 0 \tag{4.92}$$

$$v_5\lambda_5 - [L_7C_{p7}\{T_6 - (T_7 + \tau_7)\}] = 0 \tag{4.93}$$

7th effect

$$\left[\left(F_1C_{pfl}(T_f)\right)\right] - \left[\left(L_7C_{p7}(T_7 + \tau_7)\right)\right] - [(F_1 - L_7)(\lambda_7)] + [\{(L_7 - L_6) + F_V + 0.05134(L_6 - L_1 + 0.08569V_0)\}\lambda_6] = 0 \tag{4.94}$$

$$[U_7A_7(T_6 - (T_7 + \tau_7))] - [\{(L_7 - L_6) + F_V + 0.05134(L_6 - L_1 + 0.08569V_0)\}\lambda_6] = 0 \tag{4.95}$$

The model with steam splitting, variation in physical properties, vapor bleeding, feed, product and condensate flashing and fouling condition are developed and referred as Model-14 where equations are similar to that for Model-13 except that values of U are computed using Eq. 4.8 instead of Eq. 4.7.

### 4.5 SUMMARY OF ALL MODELS

A summary of the models, developed in Section 4.4, is detailed in the Table 4.3.

Table 4.3 Details of the models developed for seven effect evaporator system

| Model | Details of the model | No. of Equations |
|---|---|---|
| 1 | Model for simple seven effect evaporator system | 14 |
| 2 | Model for simple seven effect evaporator system with fouling condition | 14 |
| 3 | Model for simple seven effect evaporator system with steam splitting | 14 |
| 4 | Model for simple seven effect evaporator system with steam splitting under fouling conditions | 14 |
| 5 | Model for seven effect evaporator system with variation in physical properties and boiling point elevation | 14 |
| 6 | Model for seven effect evaporator system with variation in physical properties and boiling point elevation under fouling conditions | 14 |
| 7 | Model for seven effect evaporator system with condensate flashing | 14 |
| 8 | Model for seven effect evaporator system with condensate flashing under fouling conditions | 14 |
| 9 | Model for seven effect evaporator system with product and feed flashing | 14 |
| 10 | Model for seven effect evaporator system with product and feed flashing under fouling conditions | 14 |
| 11 | Model for seven effect evaporator system with vapor bleeding | 18 |
| 12 | Model for seven effect evaporator system with vapor bleeding under fouling conditions | 18 |
| 13 | Model for seven effect evaporator system with vapor bleeding and flashing | 18 |
| 14 | Model for seven effect evaporator system with vapor bleeding and flashing under fouling conditions | 18 |

# CHAPTER 5

## SOLUTION OF MATHEMATICAL MODELS

This Chapter deals with the solutions of mathematical models, developed in Chapter 4 for the synthesis of seven effects evaporator system operating at different conditions. The models, 1 to 14, developed in present work consist of sets of nonlinear algebraic equations. The input and output variables of these models are categorized as specified and unknown variables and are provided in Table 5.1.

Table 5.1 Specified and unknown variables for models 1 to 14

| Model | Specified variables | Unknown variables | No. of Eq. | Remark |
|---|---|---|---|---|
| 1 | $C_{p1\text{-}7}$, $\lambda_{1\text{-}7}$, $U_{c1\text{-}7}$, $A_{1\text{-}7}$, $T_7$, F, $T_f$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7,$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 2 | $C_{p1\text{-}7}$, $\lambda_{1\text{-}7}$, $U_{d1\text{-}7}$, $A_{1\text{-}7}$, $T_7$, F, $T_f$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 3 | $C_{p1\text{-}7}$, $\lambda_{1\text{-}7}$, $U_{c1\text{-}7}$, $A_{1\text{-}7}$, $T_{7,}$, F, $T_f$,$T_{01}$, $T_{02}$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 4 | $C_{p1\text{-}7}$ ,$\lambda_{01\text{-}7}$, $U_{d1\text{-}7}$, $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$, $T_{01}$, $T_{02}$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 5 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$, $T_{01}$, $T_{02}$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | Iterative method is used to find $U_{c1\text{-}7}$/ $U_{d1\text{-}7}$, $\tau_{1\text{-}7}$, $C_{p1\text{-}7}$ and $\lambda_{0\text{-}7}$ |
| 6 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$, $T_{01}$, $T_{02}$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 7 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, $V_{1v\text{-}7v}$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 8 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, $V_{1v\text{-}7v}$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 9 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, $V_{1v\text{-}7v}$, $L_i$, $F_v$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 10 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, $V_{1v\text{-}7v}$, $L_i$, $F_v$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6$ | 14 | |
| 11 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6,v_1,v_2,v_3,v_4$ | 18 | |
| 12 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6,v_1,v_2,v_3,v_4$ | 18 | |
| 13 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, $V_{1v\text{-}7v}$, $L_i$, $F_v$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6,v_1,v_2,v_3,v_4$ | 18 | |
| 14 | $A_{1\text{-}7}$, $T_{7,}$ F, $T_f$ $T_{01}$, $T_{02}$, $V_{1v\text{-}7v}$, $L_i$, $F_v$ | $V_0,L_1,L_2,L_3,L_4,L_5,L_6,L_7$ $T_1,T_2,T_3,T_4,T_5,T_6,v_1,v_2,v_3,v_4$ | 18 | |

It can be seen from Table 5.1 that the number of nonlinear equations in a model depends on the operating configuration. Further, it is observed that for these models the number of equations as well as the number of variables are equal and hence unique solution exist for all cases. The set of nonlinear algebraic equations are solved using a software called ‘system of non linear equations’.

## 5.1 ALGORITHM FOR SOLUTION OF MODELS

After careful derivation of the set of non-linear equations for each model of the system, a detailed iterative procedure is carried out to obtain the final set of solution. The set of equations are put into the ‘system of non-linear equations’ tool along with all the constants. An initial guess of overall heat transfer coefficient, U, is made for first iteration. A detailed algorithm explaining the series of steps performed for obtaining at the final solution is given as follows:

Step 1: Values of known parameters are collected from Table 3.1.

Step 2: Assuming equal temperature difference and vaporisation in each effect temperatures and liquor flow rates, respectively, are computed for each effect. An initial guess of U is made to start the calculation. For considering the model with the effect of fouling value of $R_{avg}$ is also computed along with U to predict $U_d$.

Step 3: The inclusion of variations such as steam splitting, variation in specific heat capacity, latent heat of vaporization, BPR, flashing (condensate, feed and product), and vapour bleeding are considered, if applicable. For example, solution procedure of Model-9, shown through Eqs. 4.35 to 4.40, 4.56 to 4.59 and 4.67 to 4.70, which includes flashing, is described in following steps:

Step a: Material and energy balance around each flash tank from PF1 to PF3 and SF1 to SF4 is carried out.

Step b: Balanced equations derived are mainly the functions of enthalpy of vapor and condensate and these are computed using Eq. 4.2 and 4.3. The predicted values of enthalpies are used to compute vapor flow rates emerging out from individual flash tanks using Eq. 4.52.

Step c: The values of vapor flow rates are added with the inlet vapor streams entering the steam chest of an effect.

Step 4: Set of nonlinear equations is developed based on material and energy balance around each effect, values of U, physical properties and flashed vapor flow rate.

Step 5: The set of equations are solved to obtain the revised values of temperatures and liquor flow rate of each effect using solver 'system of non linear equations'.

Step 6: Revised values of U are computed considering temperature, flow rate and concentration of each effect.

Step 7: For each effect if difference of U of two consecutive iterations falls within the range of ±30% then go to Step 8. Otherwise follow Step 3 to 7 with revised values of temperature, liquor flow rates and U.

Step 8: Steam economy is computed.

For clarity the flow chart of solution of Model-9 is shown in Fig. 5.1.

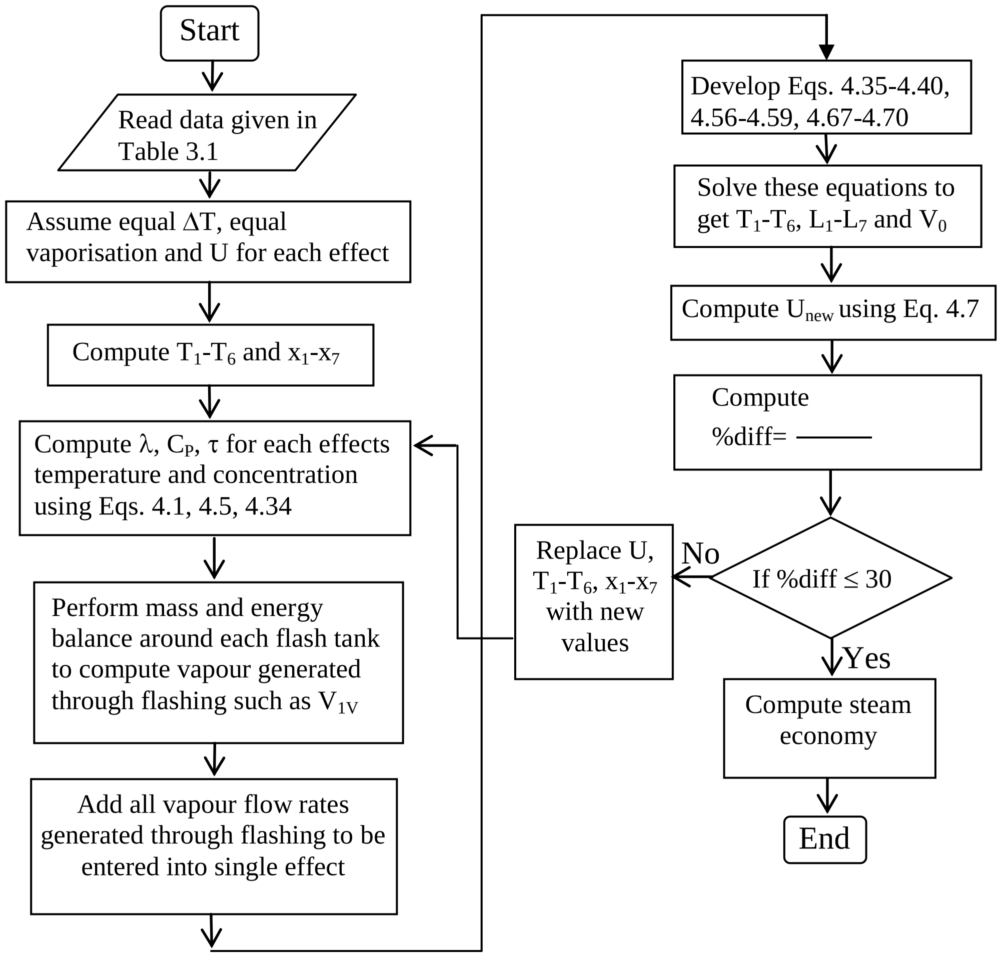



Fig. 5.1 Flow chart for solution of the Model-9

**CHAPTER 6**

# RESULTS AND DISCUSSION

The present Chapter embodies the results obtained from the theoretical investigation carried out in the present work. The MEE systems considered in this work are seven effect as well as ten effect evaporator systems. These are utilized for concentrating black liquor in typical Pulp and paper Industries as described in Chapter 3. For seven effect evaporator system 14 models are developed in Chapter 4 using different configurations such as steam splitting, variation in physical properties, vapor bleeding, feed, product and condensate flashing and fouling condition. The solution technique for models is described in Chapter 5. The sample calculation with input and output data are given in Appendix-A.

## 6.1 MODEL OF FOULING RESISTANCE

Based on the information available in the literature it is a well-known fact that seven or more number of effects is commonly used evaporator network for the concentration of black liquor in India. In such systems fouling is a major problem. As the present work deals with modeling and simulation of real MEE systems where fouling condition occurs frequently an empirical model for predicting fouling resistance, $R_{avg}$, is proposed in Section 4.3. This model is shown through Eq. 4.6 which is a function of $\Delta T$ and $V_{avg}$. Using this model value of $R_{avg}$ is computed for different effects based on $\Delta T$ and $V_{avg}$ and then added with $U_c$ through Eq. 4.8. To show results of fouling condition a case of Model-1 is considered for which values of $R_{avg}$ for seven effects are shown in Table 6.1.

Table 6.1 shows that $R_{avg}$ is found within the range from 0.1 – 0.5 $m^{2o}C/kW$. However, the resistance offered by black liquor is observed in the range of 0.1 – 1.75 $m^{2o}C/kW$ (Muller-Steinhagen and Branch, 1997). Moreover, $R_{avg}$ for same value of temperature and velocity are predicted from graphs (Muller-Steinhagen and Branch, 1997) as well as Eq. 4.6 and compared, which shows maximum difference of 0.25% between two values. Thus, the fouling resistance computed from present model compares well with the published work, which indicates the workability of the present model. The comparison of $U_c$ and $U_d$ is also presented in Table 6.1 which shows that the fouling resistance reduces value of overall heat transfer coefficient by 11.5% on average.

Table 6.1 Fouling resistance for seven effect evaporator system

| Effects | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $R_{avg}$, ($m^{2o}C/kW$) | 0.2809 | 0.4348 | 0.2119 | 0.2292 | 0.1938 | 0.1707 | 0.1459 |
| $U_c$ ($kW/m^2K$) | 0.1898 | 0.2131 | 0.5836 | 0.5624 | 0.6921 | 0.8170 | 1.005 |
| $U_d$ ($kW/m^2K$) | 0.1802 | 0.1951 | 0.5193 | 0.4982 | 0.6102 | 0.7170 | 0.8765 |

With or without considering fouling resistance 14 models are developed in the present work with different configurations. In this work two different cases of seven effects as well as ten effects evaporator systems are considered. However, all 14 models, shown in Chapter 4, are developed for seven effects evaporator system. Further, similar model is used to solve the ten effect evaporator system. The results predicted from these models are presented in the subsequent sections:

## 6.2 RESULTS FOR SEVEN EFFECT EVAPORATOR SYSTEM

In this section the results of 14 models developed for seven effect system are discussed and then that for ten effect system are considered.

### 6.2.1 Simple model for seven effect evaporator system

The simplest model of the present investigation is Model-1 which is derived based on assumptions shown in Section 4.4.1. The solution of this model is carried out with known values of $U_c$ and area. These values are taken from the work of Bhargava et al. (2008). Thus, in this case final concentration of product is computed. The detailed results of Model-1 are shown in Table 6.2.

Table 6.2 Results of Model-1

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $U_c$, kW/m$^2$°C | 0.1898 | 0.2131 | 0.5836 | 0.5624 | 0.6921 | 0.8170 | 1.005 |
| L, kg/s | 8.1456 | 9.4977 | 10.7301 | 11.8221 | 12.8008 | 13.6902 | 14.51109 |
| X | 0.221 | 0.1939 | 0.1767 | 0.1558 | 0.1439 | 0.1345 | 0.1269 |
| T, °C | 102.84 | 77.07 | 70.04 | 63.46 | 58.57 | 54.76 | 52 |

Table 6.2 shows that liquor gets more and more concentrated when it moves from 7$^{th}$ effect to 1$^{st}$ effect. The rate of water evaporation is 0.821 kg/s, 0.89 kg/s, 0.98 kg/s, 1.09 kg/s, 1.232 kg/s and 1.352 kg/s after 6$^{th}$, 5$^{th}$, 4$^{th}$, 3$^{rd}$, 2$^{nd}$ and 1$^{st}$ effect, respectively. The reason of such variation is due to availability of increased $\Delta T$ from 6$^{th}$ to 1$^{st}$ effect. It also depends on the value of $U_c$. Table 6.2 shows that product, $L_1$, is exiting the system with flow rate of 8.1456 kg/s which corresponds to the concentration of 0.221. For Model-1 total steam consumption is found as 1.787kg/s. Therefore, total steam economy of the system is 4.18.

The simple model with fouling condition is shown through Model-2. The results of this model are obtained in the similar manner as that of Model-1 and presented in Table 6.3. The product concentration from 1$^{st}$ effect predicted for Model-1 and Model-2 is 0.221 and 0.203, respectively, which shows that product concentration is considerably less in case of Model-2. Thus, steam economy for this model is found as 3.868. The decrement in steam economy when compared to Model-1 is 7.8% which is due to the effect of U. The comparison of U for each effect for Model-1 and Model-2 is shown in Table 6.1 which indicates that on average value of U is reduced by 11.5% when fouling resistance is considered. Thus, similar reduction is found in steam economy of Model-2 which is the obvious trend.

Table 6.3 Results of Model-2

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $U_d$, kW/m$^2$°C | 0.1802 | 0.1951 | 0.5193 | 0.4982 | 0.6102 | 0.717 | 0.8765 |
| L, kg/s | 9.074 | 10.297 | 11.395 | 12.35 | 13.194 | 13.950 | 14.640 |
| X | 0.203 | 0.1788 | 0.161 | 0.149 | 0.139 | 0.1320 | 0.12582 |
| T, ◦C | 102.7 | 76.91 | 69.8 | 63.23 | 58.41 | 54.41 | 52 |

### 6.2.2 Results of model with steam splitting

This section presents the results obtained for Model-3 which is developed considering steam splitting. The detailed derivation of this model is given in Section 4.4.2. Total steam is split equally in 1$^{st}$ and 2$^{nd}$ effects and enters these effects at 140°C and 147°C. Values of $U_c$ for each effect are taken from the work of Bhargava et al. (2008). This data is used to compute liquor flow rates and temperatures of each effect for Model-3 and results are summarized in Table 6.4.

It indicates exit liquor concentration and flow rates from 7$^{th}$ to 1$^{st}$ effect. The rate of water evaporation is 1.358 kg/s, 1.468 kg/s, 1.606 kg/s, 1.771 kg/s, 1.022 kg/s and 0.941 kg/s after 6$^{th}$, 5$^{th}$, 4$^{th}$, 3$^{rd}$, 2$^{nd}$ and 1$^{st}$ effect, respectively. It can be observed that for Model-3 total evaporation rate is 8.166 kg/s, which is 28.2% higher than that for Model-1. This rise in evaporation rate is due to larger temperature difference caused by steam splitting in first two effects. Due to excess evaporation for Model-3 in comparison to Model-1 the steam consumption is also more which is 2.42 kg/s. Consequently, steam economy for Model-3 is reduced to 4.05, which is 3.1% less in comparison to Model-1. Thus, induction of steam splitting increases evaporation rate but reduces the economy of seven effect evaporator system. This is because, for Model-3 steam is fed to the first two effects at higher temperatures of 140 and 147ºC, however, in Model-1 steam is entering only in first effect at 140°C and vapor generated in 1st effect is used in 2nd effect at 102.8°C. It is a fact that as temperature of steam/vapor decreases latent heat of vaporization increases. So, at higher temperature of 140°C and 147°C the first two effects of Model-3 gets lesser amount of heat of vaporization. It causes higher steam flowrate and consequently, higher steam consumption for Model-3 in comparison to Model-1 as total evaporation in first two effects is higher. It can be seen from Table 6.4 that the rate of evaporation has increased gradually from effect 7 to 3 and decreased from effect 1 to effect 2. This is due to a fact that in 3rd effect latent heat is supplied by the vapor streams emerging from 1st and 2nd effects ($V_1 \lambda_1 + V_2 \lambda_2$) together. Thus, combined value of $V_1$ and $V_2$ causes more evaporation and this fact is carried forward till 7th effect.

Table 6.4 Results of Model-3 and Model-4

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Model-3 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.310 | 0.281 | 0.372 | 0.688 | 0.884 | 1.093 | 1.434 |
| L, kg/s | 5.812 | 6.753 | 7.775 | 9.546 | 11.152 | 12.620 | 13.978 |
| X | 0.316 | 0.272 | 0.237 | 0.193 | 0.165 | 0.146 | 0.131 |
| T, ◦C | 114.83 | 92.742 | 81.55 | 70.88 | 62.86 | 56.57 | 52 |
| Model-4 | | | | | | | |
| $U_d$, kW/m$^{2o}$C | 0.343 | 0.269 | 0.329 | 0.617 | 0.794 | 0.979 | 1.270 |
| L, kg/s | 7.1736 | 7.9887 | 8.9154 | 10.4557 | 11.8294 | 13.0681 | 14.2013 |
| X | 0.2569 | 0.2306 | 0.2066 | 0.1762 | 0.1557 | 0.1409 | 0.1297 |
| T, ◦C | 115.01 | 92.22 | 80.96 | 70.37 | 62.52 | 56.42 | 52 |

On comparing the exit liquor concentration for Model-1 (without steam splitting) and Model-3 (with steam splitting) it is seen that an increase in concentration from 0.22 to 0.31 is found when steam splitting is included and it is due to higher steam consumption for Model-3.

The model with the inclusion of fouling is explained as Model-4 and the results obtained upon solving those set of equations are presented in Table 6.4. A comparison of exit liquor concentration from 1$^{st}$ effect predicted for Model-3 and Model-4 shows that product concentration is reduced by 18.98% in case of Model-4. Thus, steam economy for this model is found as 3.73. This decrement in steam economy in comparison to Model-3 is due to the effect of U. The comparison of U of each effect for Model-3 and Model-4 is shown in Table 6.5 which indicates that on average value of U is reduced by 8.99% when fouling resistance is considered.

Table 6.5 Fouling resistance for seven effect evaporator system for Model-3 and Model-4

| Effects | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $R_{avg}$,($m^{2o}C/kW$) | 0.3908 | 0.7989 | 0.3588 | 0.1935 | 0.1560 | 0.1320 | 0.109 |
| $U_c$ ($W/m^2K$) | 0.310 | 0.281 | 0.372 | 0.688 | 0.884 | 1.093 | 1.434 |
| $U_d$ ($W/m^2K$) | 0.349 | 0.269 | 0.329 | 0.617 | 0.794 | 0.9790 | 1.270 |

### 6.2.3 Results of model with variable physical properties and BPR

Model-5 is developed in section 4.4.3 which takes into account variations in $\lambda$, $C_p$ and $\tau$. The presence of the caustic soda in black liquor results in high BPR as the concentration increases. The values of $\lambda$, $C_p$ and $\tau$ are estimated using Eqs. 4.1, 4.5 and 4.34, respectively.

As Model-5 accounts variation in physical properties and BPR an iterative method as described in Section 5.1 is used. For this purpose initial values of temperatures and liquor flow rates are found based on assumption of equal $\Delta$T as well as vaporization in each effect. Using these values

$\lambda$, $C_p$, $\tau$ and U are computed. Values of U for each effect are obtained through Eq. 4.7. Then these predicted values are used in Eqs. 4.35 to 4.48 of Model-5. Thus, results obtained are used to compute $\lambda$, $C_p$, $\tau$ and U and follow the second iteration. In this manner all iterations are solved till the values of U in two consecutive iterations are less than 30%. For clarity results of all iterations of Model-5 are reported in Table 6.6. The iterations are carried out till the difference among two consecutive values of U fall within ±30% range which is obtained in 3$^{rd}$ iteration.

The results of final iteration shows that the rate of evaporation is 1.22 kg/s, 1.388 kg/s, 1.59 kg/s, 1.83 kg/s, 0.759 kg/s and 1.376 kg/s from 6$^{th}$ effect to 1. The BPR ($T_{vapor}$+BPR - $T_{vapor}$) in the 7$^{th}$ and 1$^{st}$ effect are 3.88 and 4.15, respectively. The raise in BPR from 7$^{th}$ to 1$^{st}$ effect is due to increase in concentration. It increases the solid content in the liquor which lowers the vapor pressure of solution and consequently increases the boiling point of solution. As concentration increases presence of solute molecules demand more heat to reach boiling state. The steam consumption for this case is 2.41kg/s, which gives product concentration and the steam economy as 0.318 and 4.07, respectively.

The model with variation in physical properties under fouling condition is presented as Model-6 in section 4.4.3. Its results are obtained by solving it iteratively as discussed in Model-5. The final results obtained in four iterations are presented in Table 6.7. It shows that product concentration for Model-6 is reduced by 8.5% in comparison to Model-6. Further, the comparison of U of each effect for Model-5 and Model-6 is shown in Table 6.8 which indicates that the average value of U amongst all effects is reduced by 9.96% for Model-6. The steam consumption for this model is 2.46 kg/s. Thus, steam economy for this model is found as 3.77, which is less than that for Model-5 due to the effect of fouling resistance on U.

Table 6.6 Results of all iterations of Model-5

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Iteration-1 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.163 | 0.318 | 0.412 | 0.604 | 0.738 | 0.867 | 1.057 |
| L, kg/s | 6.23 | 7.63 | 8.23 | 10 | 11.54 | 12.89 | 14.05 |
| x | 0.295 | 0.241 | 0.223 | 0.184 | 0.159 | 0.142 | 0.131 |
| τ, °C | 3.74 | 3.16 | 2.99 | 2.67 | 2.50 | 2.40 | 2.34 |
| T, °C | 95.45 | 124.11 | 89.96 | 77.27 | 67.15 | 58.71 | 52 |
| Iteration-2 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.305 | 0.464 | 0.359 | 0.523 | 0.659 | 0.794 | 0.996 |
| L, kg/s | 4.03 | 5.56 | 6.46 | 8.58 | 10.47 | 12.15 | 13.66 |
| x | 0.451 | 0.3320 | 0.284 | 0.214 | 0.175 | 0.151 | 0.134 |
| τ, °C | 6.06 | 4.20 | 3.62 | 2.91 | 2.61 | 2.45 | 2.36 |
| T, °C | 106.26 | 128.64 | 94.84 | 80.02 | 68.55 | 59.21 | 52 |
| %Difference of $U_c$ | -87.11 | -45.91 | -14.66 | -12.86 | -11.89 | 10.7 | 5.77 |
| Iteration-3 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.223 | 0.355 | 0.32 | 0.473 | 0.615 | 0.758 | 0.974 |
| L, kg/s | 4.16 | 5.6 | 6.57 | 8.69 | 10.56 | 12.22 | 13.69 |
| x | 0.442 | 0.328 | 0.280 | 0.211 | 0.174 | 0.1505 | 0.134 |
| τ, °C | 5.91 | 4.15 | 3.56 | 2.896 | 2.607 | 2.453 | 2.361 |
| T, °C | 117.6 | 132.19 | 98.7 | 81.85 | 69.32 | 59.42 | 52 |
| %Difference of $U_c$ | 26.88 | 23.49 | 10.86 | 9.56 | 6.67 | 4.53 | 2.20 |
| Iteration-4 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.215 | 0.342 | 0.335 | 0.499 | 0.660 | 0.823 | 1.062 |
| L, kg/s | 5.7885 | 7.1640 | 7.9237 | 9.7578 | 11.3515 | 12.7401 | 13.9671 |
| x | 0.318 | 0.257 | 0.232 | 0.188 | 0.162 | 0.144 | 0.131 |
| τ, °C | 4.15 | 3.37 | 3.65 | 3.75 | 3.81 | 3.85 | 3.88 |
| T, °C | 112.476 | 129.145 | 95.337 | 79.287 | 67.665 | 58.673 | 52 |
| %Difference of $U_c$ | 3.58 | 3.66 | -4.68 | -5.49 | -7.31 | -8.57 | 8.26 |

Table 6.7 Results of Model-6

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $U_d$, kW/m$^{2o}$C | 0.266 | 0.386 | 0.277 | 0.417 | 0.577 | 0.672 | 0.914 |
| L, kg/s | 6.3361 | 7.7022 | 8.4335 | 10.1926 | 11.6993 | 12.9784 | 14.0915 |
| x | 0.291 | 0.239 | 0.218 | 0.180 | 0.157 | 0.142 | 0.130 |
| $\tau$, °C | 3.68 | 3.14 | 2.95 | 2.65 | 2.49 | 2.40 | 2.34 |
| T, °C | 118.93 | 132.18 | 97.62 | 80.36 | 68.62 | 58.77 | 52 |

Table 6.8 Fouling resistance for seven effect evaporator system for Model-5 and Model-6

| Effects | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $R_{avg}$,(m$^{2o}$C/kW) | 0.324 | 0.247 | 0.436 | 0.286 | 0.216 | 0.176 | 0.142 |
| $U_c$ (W/m$^2$K) | 0.215 | 0.342 | 0.335 | 0.499 | 0.660 | 0.823 | 1.062 |
| $U_d$ (W/m$^2$K) | 0.266 | 0.388 | 0.277 | 0.417 | 0.577 | 0.672 | 0.914 |

### 6.2.4 Results of models with condensate flashing

Condensate leaving from an effect is flashed to lower temperature to obtain vapor that can be used as heating medium in the subsequent effects along with the vapor emerging from previous effect. This can be used as energy reduction scheme to reduce energy demand from outside and enhance steam economy of the system. In the seven effect evaporator system there are 7 condensate flash tanks placed between effects 3 and 7 as shown in Fig. 4.7. The condensate from each effect enters the respective flash tanks and the flashed vapor is used as heating medium for the next effects.

Model-7 is developed considering steam splitting, variations in physical properties and BPR and condensate flashing under Section 4.4.4. The detailed material and energy balance around each flash tank is explained through Fig. 4.8 and Eq. 4.52. Moreover, Model-8 is proposed where fouling condition is also considered along with all variations accounted for Model-7. Results of Model-7 and 8 are predicted using iterative method and summarized in Table 6.9.

Table 6.9 Results of Model-7 and 8

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Model-7 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.289 | 0.434 | 0.406 | 0.522 | 0.613 | 0.696 | 0.792 |
| L, kg/s | 4.35 | 5.57 | 6.51 | 8.21 | 10 | 11.7 | 13.35 |
| x | 0.422 | 0.330 | 0.282 | 0.224 | 0.184 | 0.157 | 0.137 |
| τ, °C | 5.57 | 4.18 | 3.59 | 3 | 2.677 | 2.49 | 2.37 |
| Vapor from cond. flashing, kg/s | | | | 0.236 | 0.185 | 0.197 | 0.163 |
| T, °C | 118.14 | 132.24 | 104.4 | 87.93 | 74.13 | 62.1 | 52 |
| Model-8 | | | | | | | |
| $U_d$, kW/m$^{2o}$C | 0.299 | 0.413 | 0.335 | 0.414 | 0.490 | 0.546 | 0.661 |
| L, kg/s | 5.18 | 6.36 | 7.16 | 8.85 | 10.51 | 12.06 | 13.54 |
| x | 0.355 | 0.289 | 0.256 | 0.207 | 0.175 | 0.152 | 0.135 |
| τ, °C | 4.52 | 3.67 | 3.32 | 2.86 | 2.61 | 2.46 | 2.36 |
| Vapor from cond. flashing, kg/s | | | | 0.2226 | 0.185 | 0.194 | 0.161 |
| T, °C | 117.91 | 131.92 | 104.41 | 83.68 | 68.35 | 54.67 | 52 |

The rate of evaporation observed for Model-7 are 1.65kg/s, 1.7kg/s, 1.79kg/s, 1.7kg/s, 0.94kg/s and 1.22 kg/s from effects 6 to 1. Thus, using condensate flashing total evaporation is increased when compared to previous models. This is due to the following fact: in previous models, Model-1 to 6, the heat available with condensate at different temperatures, which is a significant amount of heat, is not utilized in the evaporator system. In contrast to this, through flashing heat of condensate is extracted and vapor is produced at lower pressure (or temperature). This vapor stream is mixed with other vapor and used as heating medium is subsequent effects, which causes more evaporation. Table 6.9 shows that total 0.781 kg/s of vapor generated through flashing is used for evaporation along with steam as well as vapor of each effect. Thus, due to excess evaporation product concentration for Model-7 is also increased to 0.42. The steam consumption for this model is 2.12 kg/s. The steam economy is also increased up to 4.8 due to better integration of utilized heat through flashing.

The results of Model-8 are also presented in Table 6.9. The product concentration for Model-8 is reduced by 15.87% in comparison to Model-7. The comparison of U with and without fouling is also shown in Table 6.9 which indicates that on average the $U_d$ is reduced by 6.8%. Thus, steam economy for Model-8 is reduced to 4.62 whereas it is 4.8 for Model-7.

### 6.2.5 Results of model with feed, product and condensate flashing

Product and feed flashing is included along with condensate flashing to develop Model-9 under Section 4.4.5. The schematic diagram of the system is shown in Fig. 3.1 where vapor generated through product and feed flashing are used in steam chests of 4$^{th}$ and 7$^{th}$ effects, respectively. This allows optimum usage of vapor that is available in the system. Further, Model-9 is revised to include fouling condition to propose Model-10. These models are solved using iterative method discussed in Section 5.1 and predicted results are presented in Table 6.10. It shows that the rate of evaporation observed in Model-9 is 1.45kg/s, 1.51kg/s, 1.6kg/s, 1.59kg/s, 0.69kg/s, 1.17kg/s, from effects 6$^{th}$ to 1$^{st}$. Total evaporation is increased in this model due. The product concentration is 0.331, which is less than that predicted for Model-7. However, steam consumption for this model is also less i.e. 2.06kg/s in comparison to Model-7 and thus, steam economy is increased to 4.88.

Table 6.10 Results of Model-9 and 10

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Model-9 | | | | | | | |
| $U_c$, kW/m$^2$°C | 233.14 | 348.0069 | 406.393 | 487.5213 | 579.819 | 663.5157 | 774.1251 |
| L, kg/s | 5.55 | 6.72 | 7.41 | 9 | 10.6 | 12.11 | 13.56 |
| X | 0.331 | 0.27381 | 0.248313 | 0.204444 | 0.173585 | 0.151941 | 0.135693 |
| τ, °C | 4.198 | 3.499433 | 3.233188 | 2.835951 | 2.602634 | 2.461719 | 2.368253 |
| Vapor from cond. flashing, kg/s | | | | 0.2097 | 0.165 | 0.174 | 0.142303 |
| Vapor from feed & product flashing, kg/s | | | | 0.06 | | | 0.041 |
| T, °C | 115.6 | 130.66 | 103.4 | 87.11 | 73.6 | 61.9 | 52 |
| Model-10 | | | | | | | |
| $U_c$, kW/m$^2$°C | 0.291 | 0.4009 | 0.342123 | 0.404549 | 0.479904 | 0.547387 | 0.646616 |
| L, kg/s | 5.89 | 7.04 | 7.71 | 9.22 | 10.75 | 12.15 | 13.49 |
| X | 0.3124 | 0.261364 | 0.238651 | 0.199566 | 0.171163 | 0.15144 | 0.136397 |
| τ, °C | 3.951 | 3.366219 | 3.139087 | 2.796533 | 2.585934 | 2.458683 | 2.372085 |
| Vapor from cond. flashing, kg/s | | | | 0.2069 | 0.179 | 0.1898 | 0.160926 |
| Vapor from feed & product flashing, kg/s | | | | 0.09 | | | 0.031 |
| T, °C | 122.85 | 134.33 | 107.46 | 89.79 | 75.16 | 62.5 | 52 |

The reason of increment in steam economy is obviously due to more evaporation. As Model-9 consumes less steam more evaporation is caused due to availability of more vapors generated through feed, product and condensate flashing. For Model-9 total vapor generated through flashing is 0.792 kg/s, which is 1.4% more than that produced through condensate flashing in Model-7. Thus, feed and product flashing generate more vapors for evaporation. Moreover, feed flashing increases concentration of liquor that is entering into $7^{th}$ effect which produces a higher concentrated product after evaporation in subsequent effects. The product is flashed to remove amounts of solvent present to improve final concentration and also to produce flashed vapor. Thus, steam economy is increased up to 4.88 due to better integration of utilized heat through flashing. The feed and product flashing enhance when higher temperature of feed and product are considered.

Model-10 includes condensate, feed and product flashing and fouling resistance. The results of this model are summarized in Table 6.10 which shows on average values of $U_d$ that are decreased by 10.8% in comparison to $U_c$. Due to fouling, product concentration reduces by 5.74% as shown in Table 6.10. Thus, steam economy for this model is found as 4.71, which is 2.25% less than that of Model-9.

### 6.2.6 Results of model with vapor bleeding

As shown in Fig. 4.10 there are four pre heaters placed in between $3^{rd}$ and $4^{th}$ effect and so on till $7^{th}$ effect. Considering vapor bleeding Model-11 is developed and the results of this model are shown in Table 6.11. It shows that rate of evaporation is 1.31 kg/s, 1.48 kg/s, 1.67 kg/s, 1.91 kg/s, 0.79 kg/s, 1.4 kg/s, from effects 7 to 1 respectively. The bled vapor flow rates from streams $v_2$, $v_3$, $v_4$, and $v_5$ are found as 0.198, 0.160, 0.131, 0.113 kg/s, respectively. The product flow rate is 5.48 kg/s. The steam consumption is found to be 2.5 kg/s and corresponding steam economy is elevated to 5.17.

Table 6.11 Results of Model-11 and Model-12

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Model-11 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.176 | 0.298 | 0.356 | 0.522 | 0.68 | 0.851 | 0.999 |
| L, kg/s | 5.48 | 6.88 | 7.67 | 9.58 | 11.25 | 12.73 | 14.04 |
| Bled vapor flow rate, kg/s | | 0.198 | 0.160 | 0.131 | 0.113 | | |
| X | 0.49 | 0.267 | 0.2399 | 0.192 | 0.164 | 0.145 | 0.131 |
| $\tau$, °C | 4.25 | 3.43 | 3.151 | 2.738 | 2.535 | 2.418 | 2.344 |
| T, °C | 107.99 | 127.44 | 94.15 | 79 | 67.84 | 59.21 | 52 |
| Model-12 | | | | | | | |
| $U_c$, kW/m$^{2o}$C | 0.250 | 0.3669 | 0.3091 | 0.4328 | 0.5524 | 0.6753 | 0.7433 |
| L, kg/s | 5.55 | 5.935 | 6.896 | 9.007 | 10.866 | 12.503 | 13.9275 |
| Bled vapor flow rate, kg/s | | 0.178 | 0.155 | 0.141 | 0.149 | | |
| X | 0.42 | 0.310 | 0.267 | 0.204 | 0.169 | 0.147 | 0.132 |
| $\tau$, °C | 4.21 | 3.92 | 3.44 | 2.81 | 2.57 | 2.43 | 2.34 |
| T, °C | 115.22 | 130.37 | 99.1 | 82.88 | 70.58 | 60.82 | 52 |

Vapor bleeding is done to preheat liquor near to temperature of the effect it is entering into so that liquor can easily attain boiling temperature inside the effect. This can be explained by taking the example of preheater placed in between 3$^{rd}$ and 4$^{th}$ effects. The liquor stream $L_4$ coming out from 4$^{th}$ effect is pre-heated before it enters 3$^{rd}$ effect using part of vapor that is bled from the vapor stream entering the 3$^{rd}$ effect. So, liquor $L_4$ which has to attain a temperature of $T_3$ from $T_4$, is already achieving an intermediate temperature between $T_3$ and $T_4$ before it enters the 3$^{rd}$ effect. Consequently, less heat is required to liquor $L_4$ to boil inside the 3$^{rd}$ effect, which also reduces amount of heat required by vapor stream, $V_3$. This affect moves to first and second effect and thus reduce the steam consumption, which leads to an increase in the Steam economy.

Table 6.11 shows that average value of $U_c$ is 0.554 kW/m$^2$°C, which is considerably less in comparison to $U_c$ of previous models at clean condition. Further, after induction of pre-heater feed enters at a higher temperature to the effect, which also causes less $\Delta T$ also. So, overall value of $U\Delta T$ for each effect decreases. Consequently, amount of vapor required for evaporation also decreases. The overall effect is lower steam consumption and higher steam economy.

The model with the inclusion of fouling is explained as Model-12 and the results obtained are presented in Table 6.11. A comparison of product concentration predicted for Model-11 and Model-12 shows that it is reduced by 14.3% in case of Model-12. Thus, steam economy for this model is found as 4.75 due to 14.25% reduction of U on an average value.

### 6.2.7 Results of model with vapor bleeding and flashing

Table 6.12 shows the results obtained for the Model-13 and Model-14 developed in Section 4.4.7 which includes vapor bleeding as well as condensate, feed and product flashing. It is already shown in sections 6.2.5 and 6.2.6 how largely the induction of flashing and vapor bleeding affects the steam economy of the system.

The steam consumption for Model-13 is found to be 2.47 kg/sand hence the steam economy has reached up to 5.75. The reason for the increment in steam economy is discussed under Section 6.2.5 and 6.2.6. Due to consideration of fouling resistance in Model-14, the steam economy is reduced from 5.75 to 5.53.

Table 6.12 Results of Model-13 and Model-14

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Model-13 | | | | | | | |
| $U_c$, kW/m$^2$°C | 0.150 | 0.263 | 0.986 | 0.461 | 0.550 | 0.636 | 0.658 |
| L, kg/s | 2.45 | 3.74 | 4.78 | 6.89 | 8.98 | 11.01 | 12.94 |
| Bled vapor flow rate, kg/s | | 0.154 | 0.168 | 0.176 | 0.208 | | |
| X | 0.57 | 0.491 | 0.384 | 0.267 | 0.204 | 0.167 | 0.142 |
| τ, °C | 13.46 | 6.893 | 4.988 | 3.435 | 2.842 | 2.559 | 2.404 |
| Vapor from cond. flashing, kg/s | | | | 0.179 | 0.2131 | 0.2303 | 0.193 |
| Vapor from feed & product flashing, kg/s | | | | 0 | | | 0.041 |
| T, °C | 100.28 | 125.86 | 107.07 | 90.35 | 76.27 | 64.11 | 52 |
| Model-14 | | | | | | | |
| $U_c$, kW/m$^2$°C | 0.207 | 0.326 | 0.859 | 0.410 | 0.486 | 0.531 | 0.5696 |
| L, kg/s | 3.4 | 4.68 | 5.64 | 7.63 | 9.59 | 11.45 | 13.19 |
| Bled vapor flow rate, kg/s | | 0.181 | 0.185 | 0.202 | 0.218 | | |
| X | 0.541 | 0.393 | 0.326 | 0.241 | 0.191 | 0.160 | 0.139 |
| τ, °C | 7.85 | 5.09 | 4.12 | 3.16 | 2.73 | 2.51 | 2.38 |
| Vapor from cond. flashing, kg/s | | | | 0.165 | 0.204 | 0.218 | 0.194 |
| Vapor from feed & product flashing, kg/s | | | | 0 | | | 0.041 |
| T, °C | 102.74 | 126.67 | 108.41 | 91.51 | 77.35 | 64.3 | 52 |

## 6.3 COMPARISON OF ALL MODELS

Comparison of all 14 models, proposed in the present work, is shown in Table 6.13 to visualize how individual configuration is affecting the steam economy of the MEE system. The results of comparison are also shown through Fig. 6.1 for better visualization. It shows that maximum steam economy is observed for Model-13 where flashing as well as vapor bleeding are used. The reason of such high steam economy is better utilization of heat available with condensate and vapor bleeding as already explained under Section 6.2.5 and 6.2.6. The wavy behavior of plot, shown in Fig. 6.1, is due to considering a variation in the model at clean condition as well as fouling condition.

Table 6.13 Comparison of results of all models

| Model | Product concentration | Steam consumption, kg/s | Steam economy |
|---|---|---|---|
| 1 | 0.221 | 1.78 | 4.18 |
| 2 | 0.20 | 1.69 | 3.86 |
| 3 | 0.316 | 2.42 | 4.05 |
| 4 | 0.256 | 2.26 | 3.73 |
| 5 | 0.318 | 2.41 | 4.07 |
| 6 | 0.291 | 2.46 | 3.77 |
| 7 | 0.422 | 2.12 | 4.80 |
| 8 | 0.355 | 2.26 | 4.62 |
| 9 | 0.33 | 2.06 | 4.88 |
| 10 | 0.312 | 2.06 | 4.71 |
| 11 | 0.49 | 2.5 | 5.17 |
| 12 | 0.42 | 2.8 | 4.75 |
| 13 | 0.57 | 2.47 | 5.75 |
| 14 | 0.54 | 2.49 | 5.53 |

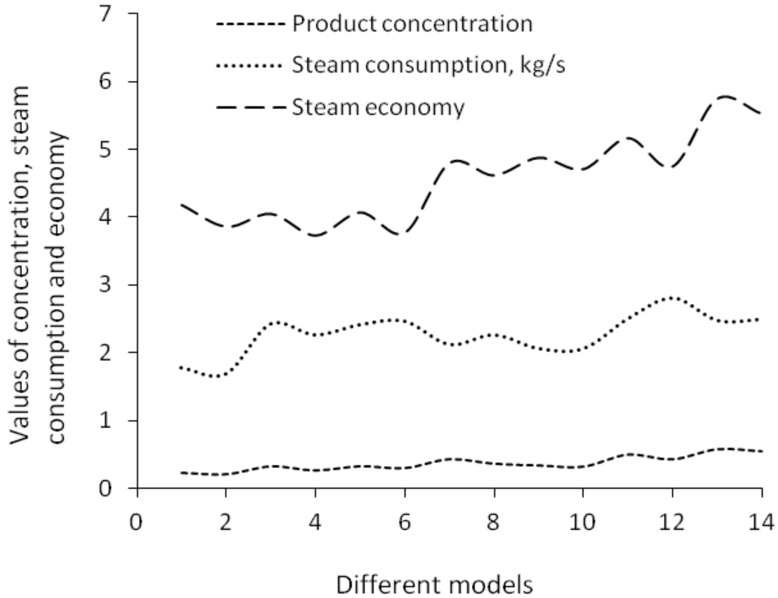
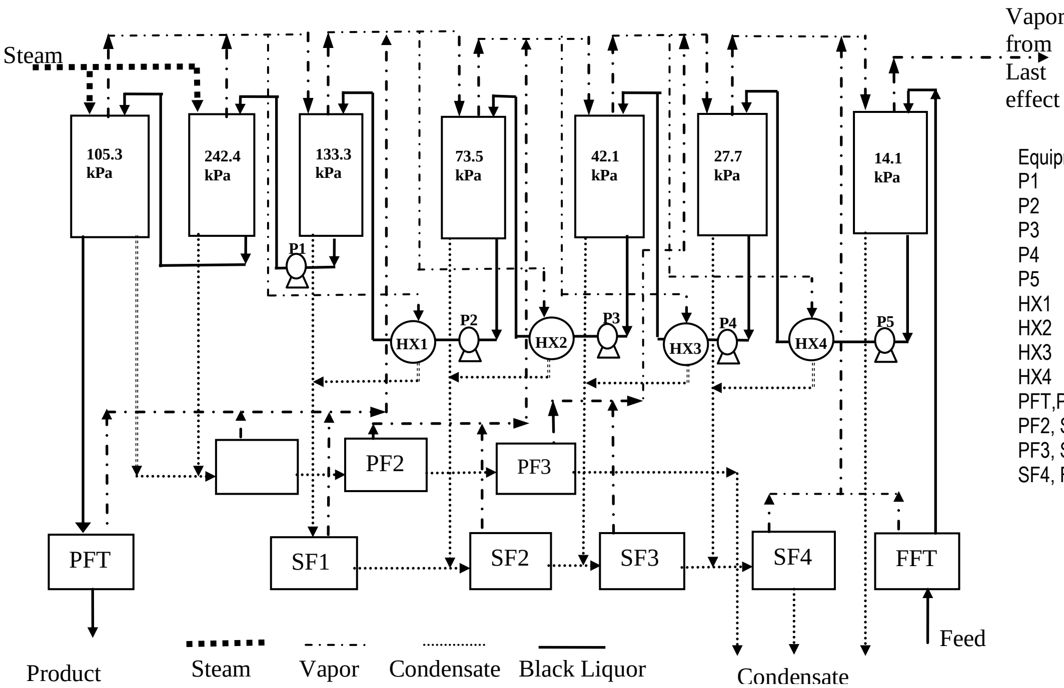



Fig. 6.1 Comparison of different models for seven effect evaporator system

Model-13 gives modified flow sheet of seven effect evaporator system, shown in Fig. 6.2, which includes four new shell and tube heat exchangers, HX1 to HX4, and is operated with backward sequence. It requires 5 new pumps, P1 to P5, also. To know the pumping requirement in the modified system pressure of each effect is computed assuming vapor is generated at saturation condition inside the effect and there is no entrained liquor in the vapor. Moreover, pressure drops across the heat exchangers are also computed using Kern's method. The power of pump is computed using pressure drops across effects as well as heat exchangers. For these pumps electricity consumption considering 8000 hours operation per year of the system is found as Rs 1.56 lakh/year. The modification, shown in Fig. 6.2, has total capital investment in terms of four heat exchangers and five pumps as Rs 29.3 lakh. However, saving in steam consumption is found as Rs 21.8 lakh/year thus, total payback period for the seven effect evaporator system, shown in Fig. 6.2, is 1.3 years.




| Equipment | Details |
|---|---|
| P1 | Power= 1.12 kW |
| P2 | Power= 1.26 kW |
| P3 | Power= 0.94 kW |
| P4 | Power= 0.86 kW |
| P5 | Power= 1.02 kW |
| HX1 | Area= 27.5 m$^2$ |
| HX2 | Area= 35.2 m$^2$ |
| HX3 | Area= 47.6 m$^2$ |
| HX4 | Area= 66.2 m$^2$ |
| PFT,PF1,SF1 | Pressure=133.3 kPa |
| PF2, SF2 | Pressure=73.5 kPa |
| PF3, SF3 | Pressure=42.1 kPa |
| SF4, FFT | Pressure=27.7 kPa |

Fig. 6.2 Schematic diagram of modified seven effect evaporator system

## 6.4 RESULTS FOR TEN EFFECT EVAPORATOR SYSTEM

This section presents the results obtained for ten effect evaporator system with mixed flow sequence explained in section 3.1.2. The model for this system is developed in a similar method followed for model development of seven effect evaporator shown in Chapter 4. This system considers all variations used in the Model-1 to Model-14 except flashing. Three preheaters are placed in between 3$^{rd}$ and 10$^{th}$ effects as shown in the Fig. 3.2 and the operating parameters are given in Table 3.2. Vapor streams are bled from $V_3$, $V_4$ and $V_6$ for preheating. A set of 23 non-linear equations are obtained for the system and results obtained after solving these set of equations are presented in Table 6.14.

Table 6.14 Results of Model of ten effect evaporator

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Clean conditions | | | | | | | | | | |
| $U_c$, kW/m$^2$°C | 0.215 | 0.150 | 0.631 | 2.863 | 1.014 | 2.078 | 0.579 | 0.713 | 0.808 | 0.396 |
| L, kg/s | 14.32 | 11.08 | 16.21 | 21.59 | 24.23 | 20.04 | 21.23 | 17.68 | 17.46 | 27.5 |
| Bled vapor flow rate, kg/s | | | 0.349 | 0.508 | | 1.271 | | | | |
| X | 0.526 | 0.680 | 0.465 | 0.349 | 0.311 | 0.376 | 0.355 | 0.426 | 0.431 | 0.274 |
| τ, °C | 7.54 | 11.26 | 6.32 | 4.43 | 3.93 | 4.83 | 4.52 | 5.63 | 5.72 | 3.50 |
| T, °C | 125.46 | 120.95 | 109.7 | 107.6 | 101.75 | 98.64 | 92 | 87.03 | 82.09 | 57.6 |
| Economy | 8.58 | | | | | | | | | |
| Fouling conditions | | | | | | | | | | |
| $U_d$, kW/m$^2$°C | 0.143 | 0.277 | 0.612 | 0.497 | 0.581 | 0.603 | 0.655 | 0.599 | 0.706 | 0.337 |
| L, kg/s | 16.29 | 14.05 | 18.33 | 23.57 | 25.98 | 20.16 | 21.58 | 18.15 | 18.08 | 29.19 |
| Bled vapor flow rate, kg/s | | | 0.331 | 0.575 | | 1.501 | | | | |
| x | 0.454 | 0.6 | 0.411 | 0.320 | 0.290 | 0.374 | 0.349 | 0.415 | 0.417 | 0.258 |
| τ, °C | 6.13 | 9.2 | 5.38 | 4.04 | 3.68 | 4.80 | 4.44 | 5.45 | 5.48 | 3.33 |
| T, °C | 130.32 | 125.81 | 112.67 | 110.42 | 103.75 | 99.61 | 92.61 | 87.42 | 82.7 | 57.6 |
| Economy | 8.00 | | | | | | | | | |

The product emerging from 2$^{nd}$ effect has a concentration of 0.68 corresponding to a flow rate of 11.08kg/s. The steam consumption is typically 4.62 kg/s which resulted in a steam economy of 8.58 when the system under clean conditions and under fouling it is found as 8.0 as shown in Table 6.14. Average $U_d$ of the system is reduced by 46.92% in comparison to $U_c$. It shows that as capacity and complexity of the system increases problem of fouling becomes more and more apparent.

## 6.5 COMPARISON OF RESULTS OF PRESENT MODEL WITH THAT OF PUBLISHED MODELS

The present models are developed for seven effect evaporator system and further, similar model is used to solve ten effect evaporator system. To check the reliability of the present study it is thought logical to compare its results with that of other investigators. The seven effect evaporator system was also simulated by Bhargava et al. (2008) and Khanam and Mohanty (2011). The comparison for results of seven effect evaporator system is shown in Fig. 6.3. It shows that using present model steam economy for seven effect system is 16.8% more than that observed by Bhargava et al. (2008). The reason is that Bhargava et al. (2008) used generalized cascade algorithm to simulate the system, however, they did not account the concept of vapor bleeding in their model. The steam economy observed by present model is 14.5% more in comparison to the results predicted by Khanam and Mohanty (2011). This is due to a fact that Khanam and Mohanty (2011) considered a number of assumptions and proposed the linear model that did not account the variation of U in the model. Thus, the model proposed by Khanam and Mohanty (2011) did not show the variations of real system and actual improvement in the system by inducting flashing, vapor bleeding, etc.

As the present two systems, seven effect and ten effect evaporator systems, are taken from the real pulp and paper industries thus, the economy of model should be compared with that of the actual system. Figs. 6.3 and 6.4 are comparing the economies of seven effect and ten effect system with industrial data. These figures show that economy is improved significantly using present model in comparison to the real systems and that is due proper use of energy reduction schemes such as flashing, bleeding.

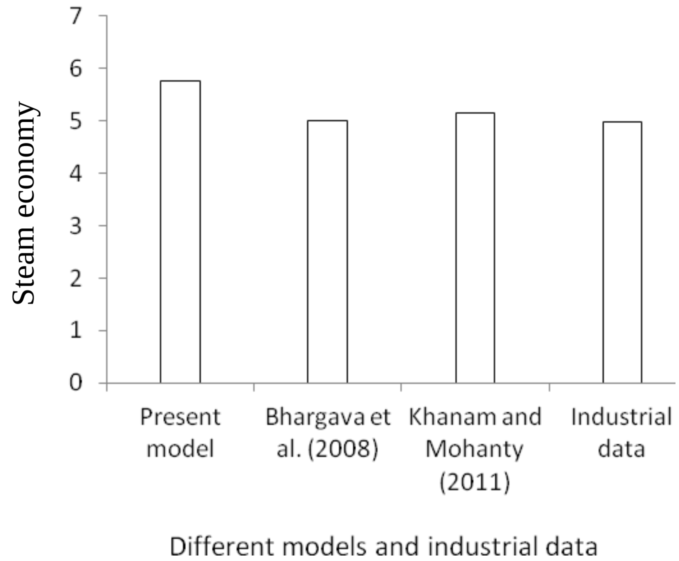



Fig. 6.3 Results of seven effect evaporator system

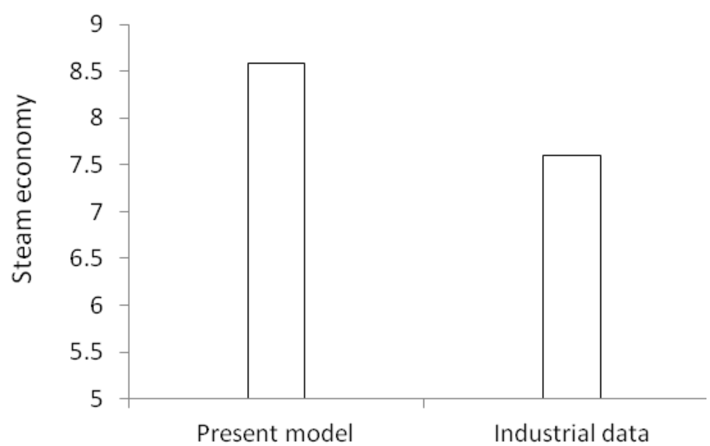



Fig. 6.4 Results of ten effect evaporator system

CHAPTER 7

# CONCLUSIONS AND RECOMMENDATIONS

In this Chapter salient conclusions drawn from the present investigations and various recommendations for future work are summarized.

## 7.1 CONCLUSIONS

This work presents a detailed study of simulation results obtained for a multiple effect evaporator (MEE) system used for concentrating black liquor through the examples of seven effect evaporator and ten effect evaporator system. By incorporating variations like steam splitting, variations in physical properties, condensate, product and feed flashing and vapor bleeding detailed models are developed which shows the variation in steam consumption and steam economy. Based on results obtained by the implementation of the present model for MEE system being operated in a nearby Pulp and Paper Industries and also by comparison of its predictions with that of published models several note worthy conclusions can be drawn. The salient conclusions are listed below:

1. The present model, based on set of nonlinear equations, addresses almost all complexities of real MEE system such as variation in physical properties, BPR, steam splitting, feed, product and condensate flashing and vapor bleeding.
2. The present work proposed a model for predicting fouling resistance offered by black liquor for MEE system based on the experimental study of Muller-Steinhagen and Branch (1997). The fouling resistance observed by the model is within the permissible limit shown in the literature.

3. The average reduction in steam economy for the MEE system is observed as 6% due to fouling condition. The fouling resistance reduces overall heat transfer coefficient by 11.5% on average.
4. Inclusion of condensate flashing along with product and feed flashing improves the steam economy of the MEE system by 16.7% where contribution of condensate flashing is 14%.
5. In the present work total 14 models are proposed for seven effect evaporator system to visualize that how individual configuration is affecting the steam economy of the MEE system. The comparison shows that maximum steam economy is observed for the Model-13 where flashing as well as vapor bleeding are used. In comparison to the simplest system, Model-1, the improvement in steam economy through best model is found as 27.3%.
6. The modified seven effect evaporator system, obtained using best model, requires four shell and tube heat exchangers and five pumps. This modification has total capital investment in terms of four heat exchangers and five pumps as Rs 29.3 lakh. However, saving in steam consumption is found as Rs 21.8 lakh/year thus, total payback period for the seven effect evaporator system is 1.3 years.
7. The vapor bleeding is applied to the case of ten effect evaporator with the same solution methodology followed for seven effect system with feed splitting and mixed flow sequence. For this system improvement in steam economy is observed as 12.8% in comparison to existing system. It incorporates three preheaters which use bled vapor from the system.

8. Based on the comparison with published model as well as industrial data it is found that the present model can be effectively applied to simulate the real MEE system and improve the steam economy of MEE system by 15%.

## 7.2 RECOMMENDATIONS

For the furtherance of knowledge in the area of simulation of MEE system, following recommendations are made:

1. In the group of flash tanks it can be seen that few flash tanks contribute substantially towards total evaporation whereas, others contribute less. This is a matter of investigation that whether flash tanks with meager contributions are economical to run or some other alternative arrangements should be employed.
2. The present model should be improved to find out optimal operating temperatures of effects of a MEE system when it is integrated with background process.

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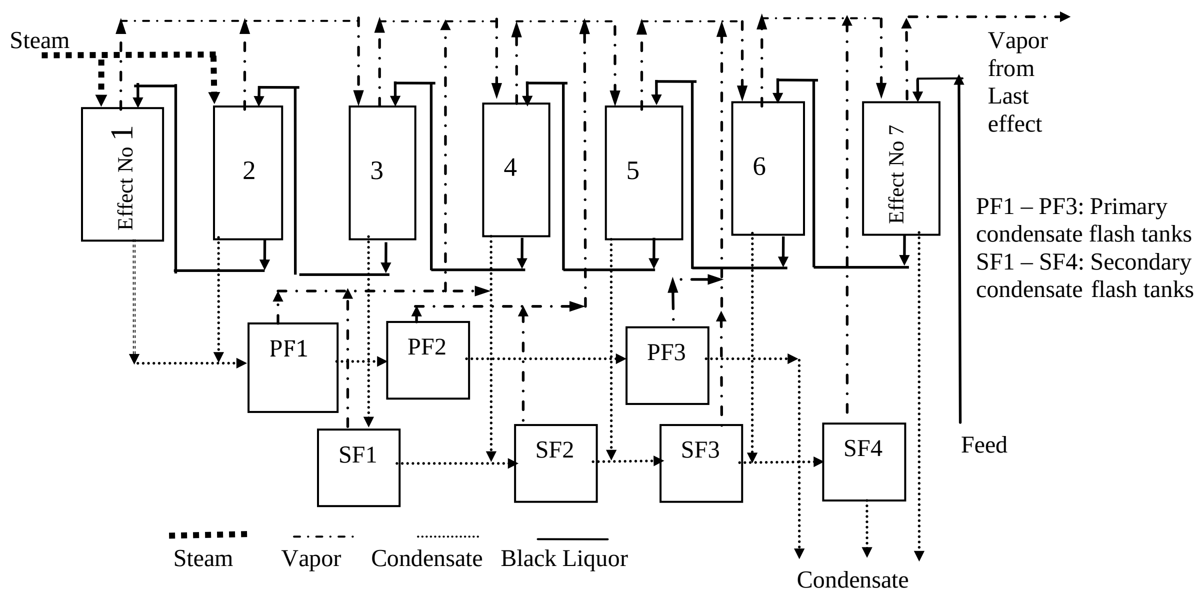
## APPENDIX A

## SAMPLE CALCULATIONS

In the present appendix sample calculations for model of seven effect evaporator system with condensate flashing are shown. The models considered in this appendix are Model-7 and 8 developed under Section 4.4.4. The step wise calculations are given below:

**Step-1**

- The figure shown below describes seven effect evaporator system containing the seven flash tanks. Model development of the system with condensate flashing is explained as Model-7. The equations developed for this model is derived in section 4.4.4. This appendix presents the step wise approach to the solution of model.




- Equations for Model-7 are reproduced below:

1st effect

$$f_1 = [L_2C_{p2}(T_2 + \tau_2)] + [0.5V_0\lambda_{01}] - [L_1C_{p1}(T_1 + \tau_1)] - [(L_2 - L_1)(\lambda_1 + (4.2T_1))]$$

$$f_2 = U_1A_1(T_{01} - T_1 - \tau_1) - (0.5V_0\lambda_{01})$$

2nd effect

$$f_3 = [L_3C_{p3}(T_3 + \tau_3)] + [0.5V_0\lambda_{02}] - [L_2C_{p2}(T_2 + \tau_2)] - [(L_3 - L_2)(\lambda_2 + (4.2T_2))]$$

$$f_4 = U_2A_2(T_{02} - T_2 - \tau_2) - (0.5V_0\lambda_{02})$$

3rd effect

$$f_5 = [L_4C_{p4}(T_4 + \tau_4)] - [L_3C_{p3}(T_3 + \tau_3)] + (L_2 - L_1)\lambda_1 + (L_3 - L_2)\lambda_2 - [(L_4 - L_3)(\lambda_3 + (4.2T_3))]$$

$$f_6 = U_3A_3\left(\frac{T_1 + T_2}{2} - T_3 - \tau_3\right) - (L_2 - L_1)\lambda_1 - (L_3 - L_2)\lambda_2$$

4th effect

$$f_7 = [L_5C_{p5}(T_5 + \tau_5)] - [L_4C_{p4}(T_4 + \tau_4)] - [(L_5 - L_4)\{\lambda_4 + (4.2T_4)\}] + [\{(L_4 - L_3) + (0.04193V_0) + 0.02832(L_2 - L_1 + L_3 - L_2)\}\lambda_3]$$

$$f_8 = [U_4A_4(T_3 - T_4 - \tau_4)] - [\{(L_4 - L_3) + (0.04193V_0) + 0.02832(L_2 - L_1 + L_3 - L_2)\}\lambda_3]$$

5th effect

$$f_9 = [L_6C_{p6}(T_6 + \tau_6)] - [L_5C_{p5}(T_5 + \tau_5)] - [(L_6 - L_5)\{\lambda_5 + (4.2T_5)\}] + [\{(L_5 - L_4) + (0.02198V_0) + 0.0229(L_2 - L_1 + L_3 - L_2 + L_4 - L_3 + 0.04193V_0)\}\lambda_4]$$

$$f_{10} = [U_5A_5(T_4 - T_5 - \tau_5)] - [\{(L_5 - L_4) + (0.02198V_0) + 0.0229(L_2 - L_1 + L_3 - L_2 + L_4 - L_3 + 0.04193V_0)\}\lambda_4]$$

6th effect

$$f_{11} = [L_7C_{p7}(T_7 + \tau_7)] - [L_6C_{p6}(T_6 + \tau_6)] - [(L_7 - L_6)\{\lambda_6 + (4.2T_6)\}] + [\{(L_6 - L_5) + (0.02177V_0) + 0.02359(L_2 - L_1 + L_3 - L_2 + L_4 - L_3 + L_5 - L_4 + 0.06392V_0)\}0.023\lambda_5]$$

$$f_{12} = [U_6A_6(T_5 - T_6 - \tau_6)] - [\{(L_6 - L_5) + (0.02177V_0) + 0.02359(L_2 - L_1 + L_3 - L_2 + L_4 - L_3 + L_5 - L_4 + 0.06392V_0)\}\lambda_6]$$

7th effect

$$f_{13} = (FC_{pf}T_f) - \{L_7C_{p7}(T_7 + \tau_7)\} - \{(F - L_7)\lambda_7\} + [\{(L_7 - L_6) + 0.05134(L_2 - L_1 + L_3 - L_2 + L_4 - L_3 + L_5 - L_4 + L_6 - L_5 + 0.08569V_0)\}\lambda_6]$$

$$f_{14} = [U_7A_7(T_6 - T_7 - \tau_7)] - [\{(L_7 - L_6) + 0.05134(L_2 - L_1 + L_3 - L_2 + L_4 - L_3 + L_5 - L_4 + L_6 - L_5 + 0.08569V_0)\}\lambda_6]$$

**Step-2**

- After deriving the equations these are solved using iterative procedure to obtain.
- Temperatures, $T_{1\text{-}7}$, vapor flow rates, $V_{1\text{-}7}$, and liquor flow rates, $L_{1\text{-}7}$, are obtained assuming equal temperature difference and equal vaporization in all effects for first iteration.

  Equal $\Delta T$ is found as

  $$\frac{T_0 - T_7}{6} = \frac{140 - 52}{6} = 14.67$$

  The value of $T_1$ to $T_7$, $V_1$ to $V_7$ and $L_1$ to $L_7$ are shown here

| | | | | | |
|---|---|---|---|---|---|
| $T_1$ | 125.33 | $V_1$ | 1.703 | $L_1$ | 3.6841 |
| $T_2$ | 131.33 | $V_2$ | 1.703 | $L_2$ | 5.3871 |
| $T_3$ | 110.66 | $V_3$ | 1.703 | $L_3$ | 7.0901 |
| $T_4$ | 96 | $V_4$ | 1.703 | $L_4$ | 8.7931 |
| $T_5$ | 81.32 | $V_5$ | 1.703 | $L_5$ | 10.4962 |
| $T_6$ | 66.65 | $V_6$ | 1.703 | $L_6$ | 12.1992 |
| $T_7$ | 52 | $V_7$ | 1.703 | $L_7$ | 13.9022 |

- Based on temperatures and flow rates of each effect their corresponding values of λ, x, $C_p$, τ are calculated and shown below:

| λ, kJ/kg | x | $C_p$, kJ/kg°C | τ, °C |
|---|---|---|---|
| 2180.409 | 0.5 | 3.05651 | 7 |
| 2160.437 | 0.342007 | 3.413728 | 4.339382 |
| 2205.879 | 0.25952 | 3.600229 | 3.347017 |
| 2240.416 | 0.209329 | 3.713712 | 2.876371 |
| 2274.068 | 0.175405 | 3.790412 | 2.615339 |
| 2347.438 | 0.150832 | 3.845972 | 2.455006 |
| 2378.189 | 0.132374 | 3.887705 | 2.350458 |

**Step-3**

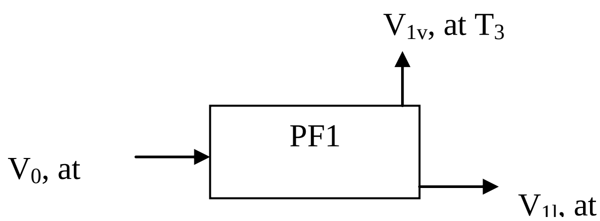
- An initial guess of overall heat transfer coefficient, $U_{c1-7}$, in W/m$^2$°C is made for the first iteration.

| | |
|---|---|
| $U_{c1}$ | 100 |
| $U_{c2}$ | 200 |
| $U_{c3}$ | 400 |
| $U_{c4}$ | 600 |
| $U_{c5}$ | 700 |
| $U_{c6}$ | 800 |

| | |
|---|---|
| $U_{c7}$ | 900 |

**Step -4**

- The vapor stream emerging from each flash tank is calculated by performing material and energy balance as shown below:




Material and energy balance around primary condensate flash tank

- Material balance $V_0 = V_{1v} + V_{1l}$
- Energy balance $V_0 h_0 = V_{1v} H_3 + V_{1l} h_3$
- flashed vapor, $V_{1v}$, from PF1 is

$$V_{1v} = V_0 \frac{(h_0 - h_3)}{(H_3 - h_3)}$$

At $T_0$ and $T_3$ enthalpies of vapor and condensate are shown in following table:

| | | | |
|---|---|---|---|
| $T_0$ | 140 | $h_0$ | 587.9886 |
| $T_3$(vapor) | 110.66 | $h_3$ | 455.048403 |
| $T_3$(condensate) | 110.66 | $H_3$ | 2688.870394 |

- Upon substitution and solving we get the flow rate of vapor stream, $V_{1v}$, emerging from PF1 as:

$V_{1v} = 0.04193V_0$

Similarly, vapor produced from six other flash tanks are predicted.

- Substituting predicted values of physical properties, BPR, $U_c$ and vapor generated through flashing in the model equations and solving these following values of 14 variables, $T_{1-7}$, $L_{1-7}$ and $V_0$, are obtained after first iteration as given below:

| | | | | | |
|---|---|---|---|---|---|
| $L_1$ | 4.91 | $T_1$ | 93.83 | $V_0$ | 1.97 |
| $L_2$ | 6.2 | $T_2$ | 123.31 | | |
| $L_3$ | 6.82 | $T_3$ | 93.83 | | |
| $L_4$ | 8.51 | $T_4$ | 76.5 | | |
| $L_5$ | 10.12 | $T_5$ | 65.44 | | |
| $L_6$ | 11.65 | $T_6$ | 55.69 | | |
| $L_7$ | 13.22 | $T_7$ | 52 | | |

- Based on the values obtained for temperatures and flow rates, shown in above table, new values of overall heat transfer coefficient, $U_{c,new}$, are calculated with the help of Eq. 4.7.

| New $U_c$, kW/m$^{2\circ}$C | |
|---|---|
| $U_{c1}$ | 158.0663 |
| $U_{c2}$ | 309.6887 |
| $U_{c3}$ | 506.5914 |
| $U_{c4}$ | 460.1939 |
| $U_{c5}$ | 674.991 |
| $U_{c6}$ | 762.9429 |
| $U_{c7}$ | 1693.014 |

- Flowrates of vapor streams emerging from all the seven flash tanks are:

| | |
|---|---|
| $V_{1V}$ | 0.16842 |
| $V_{2V}$ | 0.05669 |
| $V_{3V}$ | 0.03462 |
| $V_{4V}$ | 0.05213 |
| $V_{5V}$ | 0.11858 |
| $V_{6V}$ | 0.10785 |
| $V_{7V}$ | 0.12120 |

**Step-5**

- The values of $U_{c,new}$ and initial $U_c$ are compared. If the difference is within ±30% the iteration is stopped, otherwise new set of values are calculated based on those obtained in the preceding iteration.

| Initial $U_c$, kW/m$^{2o}$C | | $U_c$ obtained in iteration 1 | | % error |
|---|---|---|---|---|
| $U_{c1}$ | 100 | $U_{c1}$ | 158.0663 | 36.73541 |
| $U_{c2}$ | 200 | $U_{c2}$ | 309.6887 | 35.41901 |
| $U_{c3}$ | 400 | $U_{c3}$ | 506.5914 | 21.04089 |
| $U_{c4}$ | 600 | $U_{c4}$ | 460.1939 | -30.3798 |
| $U_{c5}$ | 700 | $U_{c5}$ | 674.991 | -3.70509 |
| $U_{c6}$ | 800 | $U_{c6}$ | 762.9429 | -4.85713 |
| $U_{c7}$ | 900 | $U_{c7}$ | 1693.014 | 46.84036 |

Results of all iterations for Model-7 are summarized as below

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Iteration -2 | | | | | | | |
| $U_c$, kW/m$^2$°C | 0.1480 | 0.2854 | 0.4775 | 0.3938 | 0.5931 | 0.6169 | 1.4258 |
| L, kg/s | 6.2 | 7.46 | 7.96 | 9.49 | 10.9 | 12.2 | 13.52 |
| X | 0.2967 | 0.2466 | 0.2311 | 0.1938 | 0.1688 | 0.1508 | 0.1360 |
| τ, °C | 3.761 | 3.216713 | 3.06866 | 2.751853 | 2.569918 | 2.454931 | 2.370435 |
| Vapor from flashing, kg/s | | | | 0.19 | 0.1688 | 0.13 | 0.1205 |
| T, °C | 96.23 | 124.22 | 96.23 | 77.89 | 66.67 | 55.78 | 52 |
| % error | 34.47 | 33.093 | 23.155 | -34.3 | -2.166 | 4.211 | 47.40 |
| Iteration 3 | | | | | | | |
| $U_c$, kW/m$^2$°C | 0.263 | 0.396332 | 0.395827 | 0.36859 | 0.506057 | 0.550091 | 1.83152 |
| L, kg/s | 4.7 | 5.95 | 6.7 | 8.39 | 10.05 | 11.65 | 13.32 |
| X | 0.391 | 0.309244 | 0.274627 | 0.219309 | 0.183085 | 0.15794 | 0.138138 |
| τ, °C | 5.065 | 3.912633 | 3.508398 | 2.961926 | 2.670399 | 2.4989 | 2.381643 |
| Vapor from flashing, kg/s | | | | 0.2278 | 0.2105 | 0.184 | 0.1607 |
| T, °C | 106.57 | 128.57 | 100.33 | 80.77 | 67.22 | 54.7 | 52 |
| %error | 43.81 | 27.97073 | -20.6554 | -6.86527 | -17.2101 | -12.1603 | 22.14631 |
| Iteration 4 | | | | | | | |
| $U_c$, kW/m$^2$°C | 216.49 | 329.0554 | 403.7744 | 451.6742 | 562.4734 | 643.5978 | 2231.281 |
| L, kg/s | 5.18 | 6.38 | 7.07 | 8.68 | 10.24 | 11.74 | 13.36 |
| X | 0.3552 | 0.288401 | 0.260255 | 0.211982 | 0.179688 | 0.156729 | 0.137725 |
| τ, °C | 4.523 | 3.663506 | 3.354649 | 2.898724 | 2.645752 | 2.49128 | 2.379361 |
| Vapor from flashing, kg/s | | | | 0.2309 | 0.1814 | 0.179 | 0.1466 |
| T, °C | 108.81 | 127.53 | 98.45 | 80.65 | 66.71 | 54.61 | 52 |
| %error | 25.21 | 18.77 | 2.57 | 6.14 | 2.82 | 2.81 | 0.68 |

Similar procedure is followed for solving Model-8 which takes into account the fouling rate $R_{avg}$ (computed through Eq. 4.6) along with $U_c$. The overall heat transfer coefficient including the fouling rate can be computed using Eq. 4.8.

A sample calculation is shown as below:

For first iteration values of temperatures, vapor flow rates, liquor flow rates, $\lambda$, x, $C_p$ and $\tau$ are computed in similar manner as carried out for Model-7. However, values of $U_d$ for all effects are computed using guessed values of $U_c$ in Eq. 4.8 as shown in the following table:

| Initial $U_c$ | | $1/U_c$ | $R_{avg}$ | $1/U_d$ | $U_d$ |
|---|---|---|---|---|---|
| $U_{c1}$ | 100 | 10 | 0.220716 | 10.22072 | 0.097841 |
| $U_{c2}$ | 200 | 5 | 0.220713 | 5.220713 | 0.191545 |
| $U_{c3}$ | 400 | 2.5 | 0.220713 | 2.720713 | 0.367551 |
| $U_{c4}$ | 600 | 1.666667 | 0.220711 | 1.887378 | 0.529836 |
| $U_{c5}$ | 700 | 1.428571 | 0.220709 | 1.64928 | 0.606325 |
| $U_{c6}$ | 800 | 1.25 | 0.440921 | 1.690921 | 0.591394 |
| $U_{c7}$ | 900 | 1.111111 | 0.220706 | 1.331817 | 0.750854 |

These values of $U_d$ is used to carry out the iterations till the solution converges as done for Model-7. The results for all iterations for Model-8 are summarized below:

| Effect | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Iteration 1 | | | | | | | |
| $U_d$, kW/m$^{2o}$C | 148.03 | 285.475 | 477.586 | 393.894 | 593.148 | 616.982 | 1425.896 |
| L, kg/s | 6.2 | 7.46 | 7.96 | 9.49 | 10.9 | 12.2 | 13.52 |
| X | 0.2967 | 0.246649 | 0.231156 | 0.193888 | 0.168807 | 0.15082 | 0.136095 |
| τ, °C | 3.761 | 3.216713 | 3.06866 | 2.751853 | 2.569918 | 2.454931 | 2.370435 |
| Vapor from flashing, kg/s | | | | 0.1911 | 0.1693 | 0.1307 | 0.120547 |
| T, °C | 96.23 | 124.22 | 96.23 | 77.89 | 66.67 | 55.78 | 52 |
| %error | 34.47 | 33.09397 | 23.15516 | -34.3 | -2.16673 | 4.211179 | 47.4015 |
| Iteration 2 | | | | | | | |
| $U_d$, kW/m$^{2o}$C | 263.47 | 396.33 | 395.827 | 368.59 | 506.057 | 550.091 | 1831.52 |
| L, kg/s | 4.7 | 5.95 | 6.7 | 8.39 | 10.05 | 11.65 | 13.32 |
| X | 0.391 | 0.309244 | 0.274627 | 0.219309 | 0.183085 | 0.15794 | 0.138138 |
| τ, °C | 5.06 | 3.912633 | 3.508398 | 2.961926 | 2.670399 | 2.4989 | 2.381643 |
| Vapor from flashing, kg/s | | | | 0.2278 | 0.2105 | 0.184 | 0.160700 |
| T, °C | 106.57 | 128.57 | 100.33 | 80.77 | 67.22 | 54.7 | 52 |
| %error | 43.81 | 27.97073 | -20.6554 | -6.86527 | -17.2101 | -12.1603 | 22.14631 |
| Iteration 3 | | | | | | | |
| $U_d$, kW/m$^{2o}$C | 238.11 | 345.913 | 345.075 | 351.48 | 460.001 | 512.631 | 1846.503 |
| L, kg/s | 4.78 | 5.99 | 6.78 | 8.47 | 10.09 | 11.64 | 13.3 |
| X | 0.384 | 0.307179 | 0.271386 | 0.217237 | 0.182359 | 0.158076 | 0.138346 |
| τ, °C | 4.96 | 3.887 | 3.473 | 2.943 | 2.665 | 2.499 | 2.382 |
| Vapor from flashing, kg/s | | | | 0.2269 | 0.225 | 0.2073 | 0.17342 |
| T, °C | 117.91 | 131.92 | 104.41 | 83.63 | 68.35 | 54.67 | 52 |
| %error | -10.64 | -14.5755 | -14.7073 | -4.86784 | -10.0119 | -7.3074 | 0.811411 |

**APPENDIX B**

## LIST OF PUBLICATIONS

1. A. Kumar, G. Gautami and S. Khanam, "Hydrogen distribution in the refinery using mathematical modeling", Energy, 35, 9, 2010, pp. 3763-3772.
2. G. Gautami and S. Khanam, "Study of physical properties in multiple effect evaporator system", RACET-2011, IIChE Kochi Regional Centre, March, 10-12, 2011, Cochin, Kerela. Code: RETP – 09.
3. G. Gautami and S. Khanam**,** "Selection of optimum configuration for multiple effect evaporator system" Desalination.
4. G. Gautami and S. Khanam**,** "Modeling and simulation of multiple effect evaporator" International Journal of Research and Reviews in Applied Sciences, (Accepted).